EECS 1019 – Practice 1

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Not to be handed in.

1. Multiple Choice. Which sentence relates best to the given English?

   (a) Lumber, together with marijuana, are big exports: a) $p \wedge q$; b) $p \vee q$; c) $p \oplus q$; d) other

   (b) x: a) T/F variable; b) T/F sentence; c) object; d) other

   (c) α: a) T/F variable; b) T/F sentence; c) object; d) other

   (d) Contrapositive of $p \rightarrow q$: a) $\neg p \rightarrow \neg q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; d) a & b; e) other

   (e) p is sufficient for q: a) $p \rightarrow q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) other

   (f) p is great with q: a) $p \rightarrow q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) other

   (g) p only if q. I read this one as a threat "You can have desert only if you eat your spinach." Which answer feels the most like this threat? a) $p \rightarrow q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) other

   (h) Some: a) $\forall x$ I-can-have(x); b) $\exists x$ I-can-have(x); c) other

   (i) Can I have any?: a) $\forall x$ I-can-have(x); b) $\exists x$ I-can-have(x); c) other

   (j) Everyone is married: a) $\forall x\ \exists y\ loves(x,y)$; b) $\exists y\ \forall x\ loves(x,y)$; c) other

   (k) Every real has an inverse: a) $\forall x\ \exists y$; b) $\exists y\ \forall x$; c) other

   (l) Every Christian has God: a) $\forall x\ \exists y$; b) $\exists y\ \forall x$; c) other

   (m) Jeff is not alone in that: a) $would(\text{Jeff}) \wedge \forall x \neq \text{Jeff}\ \neg would(x)$;
   b) $would(\text{Jeff}) \wedge \exists x \neq \text{Jeff}\ \neg would(x)$; c) $would(\text{Jeff}) \wedge \exists x \neq \text{Jeff}\ would(x)$; d) other

2. The goal is to translate any truth table for a Boolean formula/sentence into Disjunctive Normal Form (DNF).
   Such a sentence is the $\vee$/OR of many clauses.
   Each such clause is the $\wedge$/AND of many literals.
   Each such literal is either a variable or its negation.
   Eg. $(A \wedge \neg B \wedge \neg C) \vee (B \wedge E \wedge F)$.

   (a) Each row of the truth table, gives the evaluation of the sentence under a given an assignment A. Such an assignment gives T/F value to each of the variables. Construct a clause that says "The variables have assignment A". Denote this clause with $clause(A)$. For example, what would clause $clause(A)$ be that is equivalent to stating the assignment is $A = \langle (p_1 = T) \wedge (p_2 = F) \wedge (p_3 = T) \rangle$?

   (b) Given a truth table for sentence S, let $S_T = \{A \mid A \text{ satisfies } S\}$ be the set of assignments A under which formula S evaluates to be true, i.e., the assignment could be A_1 OR A_2 OR .... Here each satisfying assignments in S_T is listed. For example, $S = p_1 \oplus p_2$ is satisfied iff exactly one of the variables is true, i.e., the assignment is $A_1 = \langle (p_1 = T) \wedge (p_2 = F) \rangle$ or is $A_2 = \langle (p_1 = F) \wedge (p_2 = T) \rangle$. Explain how to form a DNF expression for a general sentence S. For example, what is it for the specific sentence $S = p_1 \oplus p_2$.

   (c) Consider $p_1 \oplus p_2 \oplus p_3 \oplus \ldots \oplus p_n$. For which of the T/F assignments is this true? What is this sentence called?

   (d) How many clauses would its full DNF have?

   (e) Consider the equivalence $(\alpha \wedge p) \vee (\alpha \wedge \neg p) \equiv \alpha$. It collapses the two clauses into one with the variable p removed. Note how if α is satisfied, then the variable p can flip between T and F. Use the rules in the purple table to prove the

   (f) Suppose there are two satisfying assignments/clauses that are the same for all variables, except the value of one of the variables is flipped. For example, suppose the sentence S is satisfied with both the assignment $A_1 = \langle (p_1 = T) \wedge (p_2 = F) \wedge (p_3 = T) \rangle$ and $A_2 = \langle (p_1 = T) \wedge (p_2 = F) \wedge (p_3 = F) \rangle$. How can you use the previous question to collapse/merge these into one equivalent clause?

   (g) Suppose the sentence S is $p_1 \vee p_2$. It has three satisfying assignments, $A_1 = \langle (p_1 = T) \wedge (p_2 = T) \rangle$, $A_2 = \langle (p_1 = T) \wedge (p_2 = F) \rangle$, and $A_3 = \langle (p_1 = F) \wedge (p_2 = T) \rangle$. What do these clauses merge into? Hint: one clause can merge with more than one other clause.

(h) Suppose you have an assignment A that satisfies sentence $S = p_1 \oplus p_2 \oplus p_3 \oplus \ldots \oplus p_n$. If you keep all the variables fixed except for one, and flip the value of the remaining variable, what happens to the resulting value of S? Can any of the clauses of S collapse?

3. Proofs using Purple table:

   (a) Your mother insists that you either put out the garbage or do the dishes. You convince her that you have put out the garbage and run out the door. State the rule used and its name (in purple table or in book)

   (b) If you put out the garbage, your mother will be happy. If you do the dishes, your mother will be happy. Prove that if (you put out the garbage or you do the dishes), your mother will be happy. Prove this about *garbage*, *dishes* and *happy*.
   Hint: My proof uses two rules in the purple table and 11 lines.

4. Let A denote the sentence $\exists x \forall y P(x, y)$ and let B denote $\exists y \forall x \neg P(x, y)$.
   Our goal is to either construct a P for which for which both are true,
   or to prove that for every evaluation of the relation P, they can't both be true.

   (a) As one does for "proof by contradiction", lets start by assuming that A, i.e., $\exists x \forall y P(x, y)$ is true.
   Let's determine what this says about P.
   Assume that you have an oracle A that assures you that it is true.
   Remember Jeff's oracle game.
   - What are you allowed to give her? What then does she assure?
   - What will she give you? What then does she assure?
   Use the notation $c_{\langle A,\forall\rangle}$ and $c_{\langle A,\exists\rangle}$.

   (b) Fill in as much of the first table that you know.
   In the first row and column, put in the names of the objects $c_{\langle A,\forall\rangle}/c_{\langle A,\exists\rangle}$ that you know.

A	x_1			
y_1				

B	x_1			
y_1				

   (c) Now assume that you have an oracle B that assures you that $\exists y \forall x \neg P(x, y)$ is true.
   Repeat the previous two questions. Use the second table this time.

   (d) Can both of these statements be true at the same time? Yes or No?
   If Yes, your tables should be showing such an example.
   If No, reveal to us a contradiction.
   Use the oracles again. Give and receive objects from them, until oracle A assures us of some fact and oracle B assures us of a different fact and these contradict each other, i.e., $\beta \wedge \neg\beta$.

   (e) Redo your previous proof more formally without mention of oracles.