COSC 6111 Advanced Design and Analysis of Algorithms
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Assignment: FFT

First Person:
Family Name:
Given Name:
Student #:
Email:

Second Person:
Family Name:
Given Name:
Student #:
Email:

Problem Name	If Done Old Mark	Check if to be Marked	New Mark
1 Orthogonal			

1. Orthogonal

   (a) Suppose that in your field $\omega = e^{i2\pi\frac{1}{n}}$ is an n^{th} root of unity, i.e. that $\omega^n = 1$ and k is not zero mod n. Prove that the sum $\sum_{j=0}^{n-1} \omega^{jk}$ equals zero. This is more obviously true when n is even, but we want it proved when n is odd as well. Hint: Prove and use the standard evaluation of geometric sums.

   (b) The f^{th} complex FT basis is $B_f[j] = e^{i2\pi f\frac{j}{n}}$ for $f, j \in [0, n-1]$. Use the previous answer to prove that for integers $f \neq g$ that B_f is orthogonal to B_g because $B_f \cdot B_g = \sum_{j=0..n-1} B_f[j] \times B_g[j] = 0$.

   (c) Amusingly, what is the length of the vector $|B_f|^2 = \sum_{j=0..n-1} (B_f[j])^2$. What if $f = \frac{n}{2}$?

   (d) Use the fact that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ and $e^{-i\theta} = \cos(\theta) - i\sin(\theta)$, to express $\cos(\theta)$ in terms of $e^{i\theta}$ and $e^{-i\theta}$.

   (e) Prove that for integers $f \neq g$ and $f + g \neq n$, that c_f is orthogonal to c_g because $c_f \cdot c_g = \sum_{j=0..n-1} cos(2\pi f\frac{j}{n}) \times cos(2\pi g\frac{j}{n}) = 0$.

   (f) Prove that for integers $f \neq 0$ that $|c_f|^2 = \sum_{j=0..n-1} cos(2\pi f\frac{j}{n})^2 = \frac{n}{2}$.

   (g) Think of $B_{\langle f,j\rangle}$ and $\langle c, s\rangle_{\langle f,j\rangle}$ as matrices. How do you use the above facts to compute their inverses?