York University
CSE 2001 Fall 2017 – Midterm E
Instructor: Jeff Edmonds

Family Name: _____ Given Name: _____

Student #: _____ Email: _____

1) Language	6	
2) Short	6	
3) Logic	8	
4) TM	20	
5) DFA	20	
6) Bumping	20	
7) CFG	20	
0) Art	2	
Total	102 marks	

This exam is designed to be completed in 1.5 hours, but we will give you two hours.
Keep your answers short and clear.

0) (2 marks) Art therapy question: When half done the exam, draw a picture of how you are feeling.

1. Consider the alphabet $\Sigma = \{a, b\}$. Describe in English a property defining which strings are in the language $\Sigma^* a$.

2. Short Answers: What does the following mean, is it true, and very quickly why?

   - "The regular languages are closed under intersection"

   - "Every subset of a regular language is regular."

3. First Order Logic: We state as follows that a Universal TM is a TM $M_{universal}$ is given a description of a TM M and its input I and simulates this computation.

   $\exists$ a TM $M_{universal}$, $\forall$ inputs $I_{universal} = \langle M, I \rangle$, $M_{universal}(M, I) = M(I)$.

   How is the *First Order Logic Game* game played for this statement?

   Explain how this game states whether $M_{universal}$ is likely to have more states or fewer than M.

4. **Turing Machine:** (25 marks) Write all the transition rules for a Turing Machine that solves the computational problem that takes as input a string of characters from $\{0, 1, ..., 9\}$ and replaces any occurrence of 666 with 667. Longer blocks of 6 are not changed.

   Input: 943666982796666293666
   Output: 943667982796666293667

   Pseudo Code:

```
0: Put head on first character.
   c = getCharAtHead()
   ♯6 = 0
1: loop      ⟨loop−invariant⟩: The variable ♯6 holds the number of 6 in the continuous block_1
                 to the left of the head and that we are looking at a new character c. Previous
                 changes have been made.
       if( ♯6 = 3 and c ≠ 6)   then we have see 666
            Go back to change 666 to 667
            i.e. move left back to char 6 and go to line 2
       elseif( c =blank )
            Halt
       elseif( c = 6 )              Current block of 6 has another
            ♯6 = min(♯6 + 1, 4)
       else then current block of 6 has ended
            ♯6 = 0
       Move right to next character
       c = getCharAtHead()
   end loop
2: Subroutine: Change 666 to 667
       writing a 7
       move right back to character we were just on
       ♯6 = 0
       goto line 1
```

   (a) Translate this code into Turing Machine Transitions.

   (b) How many states does your TM have?

5. DFA: Draw a DFA for the following language.
   $L = \{\alpha \in \{0,1\} \mid \alpha$ contains the substring 01 an even number of times $\}$.
   Example: 110$\underline{01}$100$\underline{01}$100 contains 01 twice, two is even, and hence the string is in the language. 1100 does not contain it, zero is even, and hence the string is in the language.
   Be sure to label the states of your DFA with names indicating what the DFA "knows" when in that state.

6. Bumping Lemma is stated as follows:
   [$\exists$ infinite set S, $\forall \alpha, \beta \in S$ with $\alpha \neq \beta$, $\exists \zeta$, $L(\alpha\zeta) \neq L(\beta\zeta)$] $\Rightarrow$ [$\forall$ DFA M, $\exists$ an input I, $M(I) \neq L(I)$].

   (a) No need for a full proof, but give reasonable intuition to why it is true.

   (b) For each of these languages, either prove that it is not regular or prove that it is. Give the intuition.

      i. $L = \{a, b\}^n \#\{u, v\}^{(n \text{ or } (n+1))}$
      ii. $L = \{a, b\}^m \#\{u, v\}^{(n \text{ or } (n+1))}$

7. For each of these languages, either give a context free grammar for it or argue that it can't be done.

   (a) $L = 0^{2m+7}\ a^n 1^{2m} \{x, y\}^s \{u, v\}^{3n+6}$

   (b) $L = 0^{2n+7}\ a^m 1^{2m} \{x, y\}^s \{u, v\}^{3n+6}$