EECS 1090 – Test 2

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1. Write in English the negation of each of these statements. Move the negation all the way. in.
   Hint: Do not trust your intuition. In your mind, translated the English provided into math, negate the math, and translate the resulting math back into English.

   (a) (5 marks) If it snows today, then I will go skiing tomorrow and have fun.

   (b) (5 marks) There is at least one person who understands every topic in mathematics.

   (c) (5 marks) In every mathematics class there is some student who falls asleep during lectures.

2. (10 marks) Express the statement "There is a student in this class that has taken some course in every department in the school of mathematical sciences" using quantifiers.
   Hint: The answer for "All students in this class love logic" would be

   $\forall s \in \text{StudentsInThisClass LoveLogic}(s)$.

3. (20 marks) Use the purple table to prove $[(K \rightarrow (M\&N))\ \&\ \neg N] \rightarrow \neg K$.
   Use deduction. Do NOT convert the $\rightarrow$ into *and* or *or*.

4. (20 marks) Use the oracle−prover−adversary game to prove the following is *valid*

   $\forall$ Models eg objects and relations

   $$[\exists x\ (G(x)\&A(x))\ \&\ \forall y\ (C(y) \rightarrow \neg G(y))] \rightarrow [\exists z(A(z)\&\neg C(z))]$$

   Hint: Follow the proof game from the slides.

   For each of Jeff's speaking bubbles have a line in your proof here.

   Hint: For each of your lines give its line number, who is talking, what they are proving, assuring, providing, or constructing, and an explanation where it comes from referring to previous line numbers.

   Eg. 3: Oracle 2 assures $\forall y\ (C(y) \rightarrow \neg G(y))$.

5. (10 marks) Prove the following is *NOT valid*

$$[\exists z(A(z)\&\neg C(z))] \rightarrow [\exists x\ (G(x)\&A(x))\ \&\ \forall y\ (C(y) \rightarrow \neg G(y))]$$

6. Easy with a twist.

   (a) (6 marks) Lets assume that every universe contains at least one object.
   Use the oracle−prover−adversary game to prove is *valid*
   $$[\forall y\ P(y)] \rightarrow [\exists x\ P(x)].$$
   Hint: One line is 2) Oracle 0 assures us that our universe contains at least one object.

   (b) (9 marks) Here is a puzzle. Suppose the universe contains no objects at all. For each of the following say whether or not it is true and why?

      i. $\forall y\ P(y)$.
      ii. $\exists x\ P(x)$
      iii. $[\forall y\ P(y)] \rightarrow [\exists x\ P(x)]$.

7. (10 marks) Suppose that $[\forall n_0,\ \exists n > n_0,\ P(n)]$ is true over the positive integers,
   i.e., an oracle assures us of it.
   Might there be exactly ten values n for which the property $P(n)$ is true?
   If not how many might there be?
   Explain. Give an example of P, eg $P(0)$ is true, $P(1)$ is false, ...