EECS 1019/1090 – Propositional Logic Practice

Instructor: Jeff Edmonds

Not to be handed in.

T is for *True*, F is for *False*. $\neg$ is *Not*, $\wedge$ is A for *And*, $\vee$ is *OR*, $\rightarrow$ is *Implies*, $\oplus$ is *Parity*,
$\forall$ is A for *forAll*, $\exists$ is E for *Exists*.

Each such rule (in the purple table) has the form: From α & β conclude γ.
- If you already have lines in your proof of the form α & β,
- then you can add the line of the form γ to your proof.

Recall, a proof is a sequence of statements, where each statement is
- either an axiom, i.e., known to be true
- or follows from previous lines using some rule from the purple table (or the book).

Number the lines of your proof 1, 2, 3, ...
For each, give the name of the rule you use to prove that line and the line numbers of any previous lines used.
Do not skip steps. (Except for dropping $\neg\neg$)
Be sure to indent appropriately.

Rule of inference	Name in Text	Name in Purple Table
p $p \rightarrow q$ $\therefore\ q$	Modus ponens	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore\ \neg p$	Modus tollens	Contra Positive + Modus Ponens
$p \rightarrow q$ $q \rightarrow r$ $\therefore\ p \rightarrow r$	Hypothetical syllogism	Transitivity
$p \vee q$ $\neg p$ $\therefore\ q$	Disjunctive syllogism	Selecting Or
p $\therefore\ p \vee q$	Addition	Eval/Building Or
$p \wedge q$ $\therefore\ p$	Simplification	Separating And
p q $\therefore\ p \wedge q$	Conjunction	Eval/Building And
$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	Resolution	
$p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore\ r$		Cases
$p \rightarrow q$ $\therefore\ \neg q \rightarrow \neg p$ $\therefore\ \neg p \vee q$		Contrapositive
$\neg(p \wedge q)$ $\therefore\ \neg p \vee \neg p$		De Morgan

1. Multiple Choice. Which sentence relates best to the given English?

   (a) Lumber, together with marijuana, are big exports: a) **(Ans)** $p \wedge q$; b) $p \vee q$; c) $p \oplus q$; d) other

   (b) Would that be fries or salad?: a) $p \wedge q$; b) $p \vee q$; c) **(Ans)** $p \oplus q$; d) other

   (c) x: a) T/F variable; b) T/F sentence; c) **(Ans)** object; d) other

   (d) p: a) **(Ans)** T/F variable; b) T/F sentence; c) object; d) other

   (e) α: a) T/F variable; b) **(Ans)** T/F sentence; c) object; d) other

   (f) Converse of $p \rightarrow q$: a) $\neg p \rightarrow \neg q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; d) **(Ans)** a & b; e) other

   (g) Contrapositive of $p \rightarrow q$: a) $\neg p \rightarrow \neg q$; b) $q \rightarrow p$; c) **(Ans)** $\neg q \rightarrow \neg p$; d) a & b; e) other

   (h) Image of $p \rightarrow q$: a) $p \rightarrow q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) **(Ans)** other

   (i) p is sufficient for q: a) **(Ans)** $p \rightarrow q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) other

   (j) p whenever q: a) $p \rightarrow q$; b) **(Ans)** $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) other

   (k) p is great with q: a) $p \rightarrow q$; b) $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) **(Ans)** other

   (l) p follows from q: a) $p \rightarrow q$; b) **(Ans)** $q \rightarrow p$; c) $\neg q \rightarrow \neg p$; e) other

   (m) p only if q. I read this one as a threat "You can have desert only if you eat your spinach." Which answer feels the most like this threat? a) $p \rightarrow q$; b) $q \rightarrow p$; c) **(Ans)** $\neg q \rightarrow \neg p$; e) other

   (n) q is necessary for p: a) $p \rightarrow q$; b) $q \rightarrow p$; c) **(Ans)** $\neg q \rightarrow \neg p$; e) other

2. Explain each of the following.

   (a) Tautology:

      i. What is the definition of the word *tautology*?

         - Answer: It is a sentence that is true under every setting of the variables.

      ii. If you made a table with a row for each T/F assignment of the variables p, r, and q, how many rows would there be.
      Hint: I don't what you to check each of them.

- Answer: There are $2^{\text{\# of variables}} = 2^3 = 8$

(b) Deduction

- Answer: Deduction:
  You say "Goal is to prove $\alpha \to \beta$ by deduction"
  You indent. Within this indenting, you assume the α.
  From this your prove the β. Then you stop the indenting and conclude: $\alpha \to \beta$.
  Reason: It is true because if α is false, them $\alpha \to \beta$ is automatically true. Hence, we only need to consider the case in which α is true. In this case, for $\alpha \to \beta$ to be true, we need β to be true.

(c) Separating And

- Answer: Separating And:
  From $\alpha \wedge \beta$, conclude α. (Conclude β too if you want.)
  Reason: It is true because $\wedge$ means that they are each separately true.

(d) Proof by cases
Hint: Our cases will be "p is true" and "p is false. What is your γ?
Hint: Be sure that you prove all three steps needed for this rule before concluding.

- Answer: Proof by cases:
  You must initially prove the following yourself:
  - 1) $\alpha \vee \beta$, i.e., at least one of the cases is true.
  - 2) $\alpha \to \gamma$, i.e., If the first case is true, you can prove γ.
  - 3) $\beta \to \gamma$, i.e., If the second case is true, you can prove γ.
  - Then you can conclude γ is true.
  Reason: It is true because, there are only two case and either way γ is true.

(e) Excluded Middle

- Answer: Excluded Middle:
  The purple table has two of these. $\alpha \vee \neg\alpha$ and $\neg(\alpha \wedge \neg\alpha)$.
  Reason: It is true because at α can't be in middle. It is either true or false, and not both

(f) Selecting Or

- Answer: Selecting Or:
  From $\alpha \vee \beta$ and $\neg\alpha$, conclude β.
  Reason: It is true because $\vee$ means that at least one of these is true. If is not α and it must be β.

(g) Building/Eval Or

- Answer: Building/Eval Or (Only needed if not used in proof):
  From α, conclude $\alpha \vee \gamma$.
  Reason: It is true because $\vee$ means that at least one of these is true. If we already know that α is true, then we are done.

3. Let p, q, s and t denote propositional (true/false) variables. Find an assignment of truth vales to p, q, s, and t that makes the following expression true. Justify your answer.
   $[p \leftrightarrow q] \;\wedge\; [(s \leftrightarrow \neg s) \to (p \vee q)] \;\wedge\; [(\neg t) \to s] \;\wedge\; [\neg(t \vee p)]$

- Answer: The AND between the clauses means that each of them needs to be true.

  Clause $\neg(t \vee p)$: De Morgan on $\neg(t \vee p)$ gives $\neg t \wedge \neg p$. Hence, we are FORCED to set $t = F$ and $p = F$.

  Clause $p \leftrightarrow q$: This states equivalence. Having $p = F$, FORCES us to set $q = F$.

  Clause $(s \leftrightarrow \neg s) \to (p \vee q)$: By "excluded middle" $(s \leftrightarrow \neg s)$ is always false. If the left hand side of an implication $\to$ is false, then the implication is true.

Clause $(\neg t) \to s$: Modus Ponens with this and $t = F$ FORCES us to set $s = T$.

In conclusion, $p = F$, $q = F$, $s = T$, and $t = F$ makes each clause true.

4. $(x = 2 \text{ or } x = 5) \to (x-2)(x-5) = 0$

   - Answer:

1)	Deduction Goal: $(x = 2 \text{ or } x = 5) \to (x-2)(x-5) = 0$	
2)	$x = 2$ or $x = 5$	Assumption/Premise
3)	Proof by Cases of $(x-2)(x-5) = 0$	
4)	Case $x = 2$:	
5)	$(x-2)(x-5) = (2-2)(2-5) = (0)(2-5) = 0$	
6)	Case $x = 5$:	
7)	$(x-2)(x-5) = (5-2)(5-5) = (5-2)(0) = 0$	
8)	$(x-2)(x-5) = 0$	Cases 2,5,7
9)	$(x = 2 \text{ or } x = 5) \to (x-2)(x-5) = 0$	Conclude deduction.

5. $[(\alpha \to \beta) \text{ and } (\neg\beta \vee \gamma)] \to [\neg\gamma \to \neg\alpha]$

   - Answer:

1)	Deduction Goal: $[(\alpha \to \beta) \text{ and } (\neg\beta \vee \gamma)] \to [\neg\gamma \to \neg\alpha]$	
2)	$(\alpha \to \beta)$ and $(\neg\beta \vee \gamma)$	Assumption/Premise
3)	$\alpha \to \beta$	Separating And 2
4)	$\neg\beta \vee \gamma$	Separating And 2
5)	Deduction Goal: $\neg\gamma \to \neg\alpha$	
6)	$\neg\gamma$	Assumption/Premise
7)	$\neg\beta$	Selecting Or 4 & 6
8)	$\neg\beta \to \neg\alpha$	Contra Positive 3
9)	$\neg\alpha$	Modus Ponens 7 & 8
10)	$\neg\gamma \to \neg\alpha$	Conclude deduction.
11)	$[(\alpha \to \beta) \text{ and } (\neg\beta \vee \gamma)] \to [\neg\gamma \to \neg\alpha]$	Conclude deduction.

6. The goal is to translate any truth table for a Boolean formula/sentence into Disjunctive Normal Form (DNF).
   Such a sentence is the $\vee$/OR of many clauses.
   Each such clause is the $\wedge$/AND of many literals.
   Each such literal is either a variable or its negation.
   Eg. $(A \wedge \neg B \wedge \neg C) \vee (B \wedge E \wedge F)$.

   (a) Each row of the truth table, gives the evaluation of the sentence under a given an assignment A. Such an assignment gives T/F value to each of the variables. Construct a clause that says "The variables have assignment A". Denote this clause with $clause(A)$. For example, what would clause $clause(A)$ be that is equivalent to stating the assignment is $A = \langle (p_1 = T) \wedge (p_2 = F) \wedge (p_3 = T) \rangle$?

   - Answer: One says $p_1 = T$ with the literal p_1. One says $p_2 = F$ with the literal $\neg p_2$. We need all of these to be true. Hence, the equivalent clause is $clause(A) = (p_1 \wedge \neg p_2 \wedge p_3)$.

   (b) Given a truth table for sentence S, let $S_T = \{A \mid A \text{ satisfies } S\}$ be the set of assignments A under which formula S evaluates to be true, i.e., the assignment could be A_1 OR A_2 OR .... Here each satisfying assignments in S_T is listed. For example, $S = p_1 \oplus p_2$ is satisfied iff exactly one of the variables is true, i.e., the assignment is $A_1 = \langle (p_1 = T) \wedge (p_2 = F) \rangle$ or is $A_2 = \langle (p_1 = F) \wedge (p_2 = T) \rangle$.

Explain how to form a DNF expression for a general sentence S. For example, what is it for the specific sentence $S = p_1 \oplus p_2$.

- Answer: The answer is the OR of these clauses, because the expression is true iff the assignment is one of these. The DNF will be $clause(A_1) \vee clause(A_2) \vee \ldots$. Here each satisfying assignments in S_T is listed. For example $(p_1 \oplus p_2) \equiv [(p_1 \wedge \neg p_2) \vee (\neg p_1 \wedge p_2)]$.

(c) Consider $p_1 \oplus p_2 \oplus p_3 \oplus \ldots \oplus p_n$. For which of the T/F assignments is this true? What is this sentence called?

- Answer: When an odd number of the variables are true. It is called Parity.

(d) How many clauses would its full DNF have?

- Answer: There are 2^n possible assignments to n variables. Half of them satisfy parity. Hence, there are $\frac{1}{2}2^n$ such clauses.

(e) Consider the equivalence $(\alpha \wedge p) \vee (\alpha \wedge \neg p) \equiv \alpha$. It collapses the two clauses into one with the variable p removed. Note how if α is satisfied, then the variable p can flip between T and F. Use the rules in the purple table to prove the

- Answer: By the distributive rule we can factor out the α,
  namely $(\alpha \wedge p) \vee (\alpha \wedge \neg p) \equiv \alpha \wedge (p \vee \neg p)$.
  By excluded middle, $(p \vee \neg p) \equiv T$.
  This gives $\alpha \vee T$, which can be simplified to α.

(f) Suppose there are two satisfying assignments/clauses that are the same for all variables, except the value of one of the variables is flipped. For example, suppose the sentence S is satisfied with both the assignment $A_1 = \langle (p_1 = T) \wedge (p_2 = F) \wedge (p_3 = T) \rangle$ and $A_2 = \langle (p_1 = T) \wedge (p_2 = F) \wedge (p_3 = F) \rangle$. How can you use the previous question to collapse/merge these into one equivalent clause?

- Answer: We merge the two clauses by keeping the partial assignment and dropping the variable that can have either value. $clause(A_1) = (p_1 \wedge \neg p_2 \wedge p_3)$, $clause(A_2) = (p_1 \wedge \neg p_2 \wedge \neg p_3)$, and $clause(A_1 \vee A_2) = (p_1 \wedge \neg p_2)$.

(g) Suppose the sentence S is $p_1 \vee p_2$. It has three satisfying assignments, $A_1 = \langle (p_1 = T) \wedge (p_2 = T) \rangle$, $A_2 = \langle (p_1 = T) \wedge (p_2 = F) \rangle$, and $A_3 = \langle (p_1 = F) \wedge (p_2 = T) \rangle$. What do these clauses merge into? Hint: one clause can merge with more than one other clause.

- Answer: $clause(A_1) = (p_1 \wedge p_2)$ and $clause(A_2) = (p_1 \wedge \neg p_2)$ collapse into simply p_1. $clause(A_1) = (p_1 \wedge p_2)$ and $clause(A_3) = (\neg p_1 \wedge p_2)$ collapse into simply p_2. The resulting sentence is the OR of the resulting clauses, namely S is $p_1 \vee p_2$. This is what we started with.

(h) Suppose you have an assignment A that satisfies sentence $S = p_1 \oplus p_2 \oplus p_3 \oplus \ldots \oplus p_n$. If you keep all the variables fixed except for one, and flip the value of the remaining variable, what happens to the resulting value of S? Can any of the clauses of S collapse?

- Answer: Being Parity, if A satisfies S then an odd number of its variables are true. If you flip one value, the number true will flip from odd to even making S false. This demonstrates that no two clause can collapse. S needs to retain all $\frac{1}{2}2^n$ of its clauses.

7. Proofs using Purple table:

(a) Your mother insists that you either put out the garbage or do the dishes. You convince her that you have put out the garbage and run out the door. State the rule used and its name (in purple table or in book)

- Answer: Building/Eval Or: From α, conclude $\alpha \vee \gamma$.
  Reason: It is true because $\vee$ means that at least one of these is true. If we already know that α is true, then we are done.
  From *garbage*, build $[garbage \vee dishes]$.

(b) If you put out the garbage, your mother will be happy. If you do the dishes, your mother will be happy. Prove that if (you put out the garbage or you do the dishes), your mother will be happy. Prove this about *garbage*, *dishes* and *happy*.
Hint: My proof uses two rules in the purple table and 11 lines.

- Answer:

1) $garbage \rightarrow happy$	Axiom
2) $dishes \rightarrow happy$	Axiom
3) Deduction Goal: $(garbage \vee dishes) \rightarrow happy$.	
4) $garbage \vee dishes$	Assumption/Premise
5) Cases Goal: *happy*. Cases *garbage* and *dishes*.	
6) Case 1: *garbage*	Assumption/Premise
7) *happy*	Modus Ponens 1 & 6 .
8) Case 2: *dishes*	Assumption/Premise
9) *happy*	Modus Ponens 2 & 8.
10) *happy*	Conclude cases 4, 7, & 9
11) $(garbage \vee dishes) \rightarrow happy$	Conclude deduction.

8. $[A \rightarrow B] \rightarrow [B \rightarrow A]$

(a) If valid, i.e.. true in every setting, what would it mean?

- Answer: It says that for every implication, the converse is also true.

(b) In class, Jeff demonstrates that this is not always true by giving a counter example involving objects in our daily life. Give that example or, if you can't remember it, make up another.

- Answer: $[Hound \rightarrow Dog] \rightarrow [Dog \rightarrow Hound]$

(c) Give an assignment to A and B under which this expression evaluates to false.
Do this by forming a tree of T/F. Under the sentence below, write T or F under each variable. Below that write T or F for each [...]. Below this write F for the entire expression.
$[A \rightarrow B] \rightarrow [B \rightarrow A]$

- Answer: Start by thinking of a setting of X and Y for which $[X \rightarrow Y]$ is false.
It is easier to do this backwards, i.e., do the last F, then for [], then for A and B.
$[A \rightarrow B] \rightarrow [B \rightarrow A]$
$[F \rightarrow T] \rightarrow [T \rightarrow F]$
$[T] \rightarrow [F]$
F

(d) There is a rule that translates $[X \rightarrow Y]$ into the OR/$\vee$ expression $[??\vee??]$.
Use this rule to translate each of the $\rightarrow$ into Or/$\vee$.
Then use other rules to put it into the easier of:
- Conjunctive Normal Form $(A \vee \neg B \vee \neg C) \wedge (\neg D \vee E \vee F)$
- Disjunctive Normal Form $(A \wedge \neg B \wedge \neg C) \vee (\neg D \wedge E \wedge F)$.
You might want to check that under the setting from the previous question, the statement is still false.

- Answer: $[A \rightarrow B] \rightarrow [B \rightarrow A]$
$[\neg A \vee B] \rightarrow [\neg B \vee A]$
$\neg[\neg A \vee B] \vee [\neg B \vee A]$
$[A \wedge \neg B] \vee \neg B \vee A$
$[F \wedge \neg T] \vee \neg T \vee F$
$F \vee F \vee F = F$

9. *Resolution* plays an important role in AI and is used in Prologue.
It's *Cut Rule* corresponds to the statement: $S \equiv [[(\neg p \vee r) \wedge (p \vee q)] \rightarrow [r \vee q]]$.

(a) Prove S.
Hint: My proof uses the following rules in the given order. (a) Deduction (b) Separating And (c) Proof by cases (d) Excluded Middle (e) Selecting Or (f) Building/Eval Or
Hint: My proof has 14 lines.
Hint: Our cases will be "p is true" and "p is false. What is your γ?

- Answer:

1)	Deduction Goal: $[(\neg p \vee r) \wedge (p \vee q)] \rightarrow [r \vee q]$	
2)	$(\neg p \vee r) \wedge (p \vee q)$	Assumption/Premise
3)	$\neg p \vee r$	Separating And
4)	$p \vee q$	
5)	Cases Goal: $r \vee q$. Cases p and $\neg p$.	
6)	$p \vee \neg p$	Excluded Middle
7)	Case p:	Assumed by cases/deduction.
8)	r	Selecting Or from 3 and 7
9)	$r \vee q$	Building/Eval Or from 8
10)	Case $\neg p$:	Assumed by cases/deduction.
11)	q	Selecting Or from 4 and 10
12)	$r \vee q$	Building/Eval Or from 11
13)	$r \vee q$	Conclude cases 6, 9, & 12
14)	$[(\neg p \vee r) \wedge (p \vee q)] \rightarrow [r \vee q]$	Conclude deduction.

(b) Now that you have proved that $[(\neg p \vee r) \wedge (p \vee q)] \rightarrow [r \vee q]$ is true, you added to the Jeff's big purple table of rules. How then do you use it in a proof.

- Answer: If you have already proved $(\neg p \vee r)$ and $(p \vee q)$ as lines of your proof, then you can add $[r \vee q]$ as a line of your proof. This is called the Cut Rule.

10. Simplifying Parity (Purple Table): Suppose I tell you that α is true. I claim that this means that everywhere $\alpha \oplus \beta$ appears it can be simplified to $\neg\beta$. Let's consider whether this is true.

(a) What does $\alpha \oplus \beta$ NOT means?

i. At least one of these is true.
ii. Exactly one of these is true.
iii. We add them together in binary where $1+1=0$.
iv. $(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)$.
v. All correct meanings.

- Answer: (i)

(b) Assuming α, which of the following is NOT true?

i. $\alpha \oplus \beta$ is true iff β is false.
ii. β is true then $\alpha \oplus \beta$ is false.
iii. β must be true.
iv. $\alpha \oplus \beta$ iff $(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)$.
v. All true

- Answer: (iii)

(c) Prove $\alpha \rightarrow [(\alpha \oplus \beta) \equiv \neg\beta]$.

- Answer:

1)	Deduction Goal: $\alpha \rightarrow [(\alpha \oplus \beta) \equiv \neg\beta]$	
2)	α	Assumption/Premise
3)	Proof by Cases of $(\alpha \oplus \beta) \equiv \neg\beta$	

4) Case β is true
5) $\alpha \oplus \beta = T \oplus T = F$ Definition of $\oplus$
6) Case β is false
7) $\alpha \oplus \beta = T \oplus F = T$ Definition of $\oplus$
8) $\beta \vee \neg\beta$ Excluded Middle
9) $(\alpha \oplus \beta) \equiv \neg\beta$ Cases 5,7,8
10) $\alpha \rightarrow [(\alpha \oplus \beta) \equiv \neg\beta]$ Conclude deduction.

(d) Prove $\alpha \rightarrow [\neg\beta \rightarrow [(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)]]$.

- Answer:

1) Deduction Goal: $\alpha \rightarrow [\neg\beta \rightarrow [(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)]]$
2) α Assumption/Premise
3) Deduction Goal: $[\neg\beta \rightarrow [(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)]]$
4) $\neg\beta$ Assumption/Premise
5) $(\alpha \wedge \neg\beta)$ Building/Eval And (2 & 4)
6) $(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)]$ Building/Eval Or (5)
8) $[\neg\beta \rightarrow [(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)]]$ Conclude deduction.
9) $\alpha \rightarrow [\neg\beta \rightarrow [(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)]]$ Conclude deduction.

(e) Prove $\alpha \rightarrow [[(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)] \rightarrow \neg\beta]$.

- Answer:

1) Deduction Goal: $\alpha \rightarrow [[(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)] \rightarrow \neg\beta]$
2) α Assumption/Premise
3) Deduction Goal: $[(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)] \rightarrow \neg\beta$
4) $(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)$ Assumption/Premise

The purple table gives two ways to use an OR: "Selecting Or" and "Cases".
In "Cases", the first case would be $(\alpha \wedge \neg\beta)$ and the second $(\neg\alpha \wedge \beta)$.
The problem with doing this is that that the second case leads to a contradiction.
It is not clear what to do with that.
In "Selecting Or", we prove that the first is true by proving the second is false. Lets do that.
We know α is true. Hence, $(\neg\alpha)$ is false and so is $(\neg\alpha \wedge \beta)$.
"Building/Eval And" lets you do that, i.e. from $\neg\beta$ conclude $(\beta \wedge \gamma)$ is false.

5) $\neg(\neg\alpha \wedge \beta)$ Building/Eval And (2)

Having proved the second case is false, we know that the first is true.

6) $(\alpha \wedge \neg\beta)$ Selecting Or (4&5)

As would have happened in "Cases", the first case gives us what we want.

7) $\neg\beta$ Separating And (6)
8) $[(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)] \rightarrow \neg\beta$ Conclude deduction.
9) $\alpha \rightarrow [[(\alpha \wedge \neg\beta) \vee (\neg\alpha \wedge \beta)] \rightarrow \neg\beta]$ Conclude deduction.