EECS 1090 O – Exam (2026)

Instructor: Jeff Edmonds

1. We are going to do a reduction to prove one computational problem is a least as hard as another. Though you might want to panic, the problem is REALLY easy.
   And I have broken the problem into manageable parts.

   (a) (20 min. 16 sentences) We define the Oracles and Prover as follows.
   Hint: Read this table row by row.

Problem: Graph Colouring	**Problem:** Scheduling
Input: $\langle V, E, C\rangle$. An undirected graph with vertices V and edges E. And a set of colours C.	**Input:** $\langle U, P, T\rangle$. A set of courses U. A set of professors P indicating which courses they teach. A set of time slots T.
Output: If possible, $S_c : V \rightarrow C$ is a colouring of each vertex with one of the colours such that every edge has two different colours.	**Output:** If possible, $S_s : U \rightarrow T$ is a scheduling of each course to a time slot such that for every professor her courses are at different times.
Reduction: If we have a fast algorithm this $\Rightarrow$	we have a fast algorithm this.
Algorithm: In CS we write an algorithm that takes the input and produces the output with some guarantee that it is correct.	**Logic Game:** Our Logic game prover/oracle does the same.
Oracle 1 (Subroutine): $\forall\langle V, E, C\rangle$ $\exists$ (if possible) S_c $\forall$ edges $\{v_1, v_2\} \in E$ $S_c(v_1) \neq S_c(v_2)$	**Prover:** $\forall\langle U, P, T\rangle$ $\exists$ (if possible) S_s $\forall$ professors $p \in P$ $\forall$ pairs of courses she teaches $\{u_1, u_2\}$ $S_s(u_1) \neq S_s(u_2)$
Oracle 2 (Instance Map): $\forall\langle U, P, T\rangle$ $\exists\langle V, E, C\rangle$??	**Oracle 4 (Part of Bijection):** $\forall\{u_1, u_2\} \in ??$ $\exists\{v_1, v_2\} \in E$??
Oracle 3 (Solution Map): $\forall S_c$ $\exists S_s$??	**Oracle 5 (Miracle):** $[S_c(v_1) \neq S_c(v_2)]$ iff $[S_s(u_1) \neq S_s(u_2)]$.

   Exam Question: In the model of graphs and courses, you are to blindly play the game in order to proof the following true: Oracles 1-5 $\rightarrow$ Prover
   Hint: No thinking required. Just pass objects of the correct type to the correct people. Be sure to check that conditions are right before you pass things along.
   Hint: Do not worry about the ?? until the next question.

   - Answer: To prove $\rightarrow$ assume Oracle 1-5. Our goal as the prover is to prove Prover.
     To prove $\forall\langle U, P, T\rangle$, my adversary gives me his worst case/arbitrary instance $\langle U, P, T\rangle$ to the scheduling problem that I must solve.
     I give this $\langle U, P, T\rangle$ to Oracle 2 and she gives me $\langle V, E, C\rangle$.
     I give this $\langle V, E, C\rangle$ to Oracle 1 and she gives me a valid colouring S_c.
     I give this S_c to Oracle 3 and gives me a schedule S_s.
     This is how I construct the S_s to prove $\exists S_s$.
     Now I must guarantees that this schedule is valid.
     To prove $\forall p \in P$ $\forall\{u_1, u_2\}$ she is teaching, my adversary gives me his worst case/arbitrary pair of courses $\{u_1, u_2\}$ taught by some professor p.
     I give Oracle 4 this pair of courses $\{u_1, u_2\}$ (with some property ??) and she gives me a pair of vertices $\{v_1, v_2\} \in E$ that is an edge of the graph. I give Oracle 1 this pair of vertices $\{v_1, v_2\}$. I can do so because it is an edge of the graph. She assure me that $S_c(v_1) \neq S_c(v_2)$.
     I give Oracle 5 this assurance that $S_c(v_1) \neq S_c(v_2)$ and she assure me (by a miracle) that $S_s(u_1) \neq S_s(u_2)$.
     This is exactly what I needed to prove.
     This completes the game.

   (b) (10 min. 6 sentences) The previous question was quite generic. It did not say anything about the problems at hand. I will fill in the details of Oracle 2 (Instance Map). In the next few questions, you will have to fill out the rest.

Oracle 2: $\forall\langle U,P,T\rangle\ \exists\langle V,E,C\rangle$??
She is in a meeting in which every one is talking about this year's course scheduling $\langle U,P,T\rangle$. Being bored, she doodles. She draws a dot v for each course u. She draws a line between the dots v_1 and v_2 if some professor is teaching both of these courses. She also gets out her markers and highlights each time slot $t\in T$ with a different colour $c\in C$. Her boss sees her and fusses at her for not paying attention.

Oracle 3 (Solution Map):
$\forall S_c\ \exists S_s$??
Follows Oracle 2's bijections.

Oracle 4 (Part of Bijection):
$\forall\{u_1,u_2\}\in ??\ \exists\{v_1,v_2\}\in E$??
Follows Oracle 2's bijections.

Exam Question: We are trying to prove that though these two problems, Graph Colouring and Scheduling, look different on the surface, if you change the names for things, they really are the **same** problem. We change the names for things by looking for bijections between objects in one problem and these in the other. Oracle 2 sets up a few such bijections. Explain each of them:

i. Vertices
ii. Potential Edges (Leaving the definition of E until the next question)
iii. Colours
iv. Solutions

- Answer: We are suppose to look for bijections. If you pay a little attention to Oracle 2:
  i. Vertices: "She draws a dot v for each course u." This sets up a bijection between each course u and a vertex v.
  ii. Potential Edges (Ignoring the definition of E): A bijection between courses u and vertices v induces a natural bijection between pairs of courses $\langle u_1,u_2\rangle$ and edges $\langle v_1,v_2\rangle$.
  iii. Colours: "She also gets out her markers and highlights each time slot $t\in T$ with a different colour $c\in C$." This is also a bijection.
  iv. Solutions: If $v=u$ and $c=t$, then the solutions $S_c:V\rightarrow C$ and $S_s:U\rightarrow T$ really are the same.

(c) (6 min. 3 sentences)

Oracle 1 (Subroutine):
$\ldots\forall$ edges $\{v_1,v_2\}\in E\ldots$

Oracle 4 (Part of Bijection)'s job is
$\forall\{u_1,u_2\}\in ??\ \exists\{v_1,v_2\}\in E$??

Exam Question: When you played the logic game, you might have casually handed Oracle 1 an edge $\{v_1,v_2\}$ without not even noticing that she needs this potential edge to be part of the graph E and that Oracle 4 assures you that it is. Discuss how Oracle 4 knows that $\{v_1,v_2\}\in E$.

- Answer: When the adversary gives us a pair of courses $\langle u_1,u_2\rangle$, he assures us that there is a professor p that teaches them both. By Oracle 2's construction, "She draws a line between the dots v_1 and v_2 if some professor is teaching both of these courses." This is exactly when she adds the edge $\langle v_1,v_2\rangle$ to the graph E.

(d) (3 min. 1 sentences)

Oracle 5 (Miracle):
$[S_c(v_1)\neq S_c(v_2)]$ iff $[S_s(u_1)\neq S_s(u_2)]$.
Your last job is to prove that this is true.

- Answer: What is there to say. Under the renaming of objects the mappings S_c and S_s are the **same**.