# Oracle Solutions

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1.a)

1. Deduction Goal:
2. Assumption 1
3. Quantifer Negation 2
4. DeMorgan’s 3
5. Conclude Deduction

1.b)

1. Deduction Goal:
2. Assumption 1
3. Simplification 2
4. Remove 3
5. Simplification 2
6. Remove 5
7. Assumption
8. Modus Ponens 4,7
9. Modus Ponens 6, 8
10. Deduction 7-9
11. ∀w (L(w) → N(w)) Add 10
12. Conclude Deduction

1.c)

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. Remove 3
5. Simplification 4
6. Simplification 4
7. Simplification 2
8. Remove 7
9. Contrapositive 8
10. Modus Ponens 5, 8
11. Eval/Build 6, 10
12. Add 11
13. Conclude Deduction

1.d)

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. Quantifier Negation 3
5. DeMorgan’s 4
6. Simplification 2
7. Remove 5, 6
8. R(j) Selecting Or 6,7
9. Conclude Deduction

1.e)

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. Contrapositive 3
5. DeMorgan’s 4
6. Simplification 2
7. Remove 6
8. Remove 5
9. Modus Ponens 7, 8
10. Simplification 9
11. Add 10
12. Conclude Deduction

1.f)

1. Deduction Goal :
2. Assumption
3. ∀x F(x) Simplification 2
4. Remove 3
5. ∀z H(z) Simplification 2
6. Remove 5
7. Eval / Build 4,6
8. Double Negation 7
9. DeMorgan’s 8
10. Add 9
11. ¬∃y (¬F(y) ∨ ¬H(y) Quantifier Negation 10
12. Conclude Deduction

2. More:

(a)

1. Deduction Goal:
2. Assumption
3. Simplification 2
4. Quantifier Negation 3
5. Simplification 2
6. Contrapositive 5
7. Modus Ponens 4,6
8. Quantifier Negation 7
9. Conclude Deduction

(b)

1. Deduction Goal :
2. Assumption
3. Quantifier Negation 2
4. DeMorgan’s 3
5. Equivalence 4
6. Conclude Deduction

(c)

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. Quantifier Negation 3
5. DeMorgan’s 4
6. Quantifier Negation 5
7. Remove 6
8. Simplification 7
9. Add 8
10. Simplification 7
11. Add 10
12. Conjunction 9, 11

Conclude Deduction

**(d)**

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. DeMorgan’s 3
5. Remove 4
6. Simplification 5
7. Simplification 2
8. Remove 7
9. Contrapositive 8
10. Modus Ponens 6, 9
11. Conclude Deduction

**(e)** Note ∃x ¬K(x, x) is an error it should be ∃x ¬K(x)

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. Remove
5. Eval/Build 4
6. Simplification 2
7. Remove 7
8. Contrapositive 7
9. Modus Ponens 5,8
10. Add 9
11. Conclude Deduction

**(f)**

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. Remove 3
5. Simplification 4
6. Simplification 4
7. Simplification 2
8. Remove 7
9. Quantifier Negation 8
10. Remove 9
11. Equivalence 10
12. Contrapositive 11
13. Modus ponens 5, 11
14. Remove 6
15. Assumption
16. Remove 15
17. Modus ponens 14, 16
18. Selecting Or 13, 17
19. Deduction 16-18
20. Add 19
21. Conclude Deduction

**(g)**

1. Deduction Goal :
2. Assumption
3. Assumption
4. Remove 3
5. Simplification 4
6. Simplification 4
7. Simplification 2
8. Remove 7
9. Modus Ponens 5, 8
10. Simplification 2
11. Remove 10
12. Modus Ponens 6, 11
13. Assumption
14. Remove 13
15. Remove 9
16. Modus Ponens 14, 15
17. Modus Ponens 6, 11
18. Remove 17
19. Contrapositive 18
20. Modus Ponens 16, 19
21. Deduction 14-20
22. Add 21
23. Deduction 3-22
24. Conclude Deduction

**(h)**

1. Deduction Goal :
2. Assumption
3. ) Assumption
4. Remove 3
5. Simplification 4
6. Simplification 4
7. Remove 2
8. Modus Ponens 5, 7
9. Eval/Build 6, 8
10. Add 9
11. Deduction 3-10
12. Add 11

(i) (Solution also in slides)

1. Deduction Goal :
2. Assumption
3. Simplification 2
4. Contrapositive 3
5. Simplification 2
6. Contrapositive 3
7. Assumption
8. Assumption
9. Assumption
10. Quantifier Negation 9
11. Remove 10
12. Eval/Build 11
13. Add 12
14. Quantifier Negation 13
15. Modus Ponens 6, 14
16. Quantifier Negation 15
17. DeMorgan’s 16
19. Case 1:
20. Eval/Build 18
21. Add 20
22. Case 2:
23. Eval/Build 22
24. Add 23
25. Modus Ponens 4, 24
26. Proof by cases 19-25
27. Deduction 9-26
28. Conclude Deduction

3.a)

1. Deduction Goal:
2. Assumption
3. Equivalence 2
4. Conclude Deduction

B)

1. Deduction Goal:
2. Assumption
3. Assumption
4. Remove 2
5. Modus Ponens 3, 4
6. Modus Ponens 3, 5
7. Deduction 3, 7
8. Add 7
9. Conclude Deduction

C)

1. Deduction Goal:
2. Assumption
3. Quantifier Negation 2
4. Demorgan’s 3
5. Equivalence 4
6. Simplification 5
7. Conclude Deduction 2-6

D) (This assumes a non-empty domain, that there exists at least one *x*)

Deduction Goal:

I believe this one is false. **Counter example**: a non-empty domain such that there exists one *x* which is *B(x)*. The statement is true (*all* of the x’s that are A(x) are also B(x), which in this case are none), however is not true (there is only one entity which is *B(x)*.

E)

1. Deduction Goal:
2. Assumption
3. Assumption
4. Remove 3
5. Modus Ponens 2, 3
6. Remove 5
7. Deduction 4-6
8. Add 7
9. Conclude Deduction

F) This one is interesting because it looks like the example from the slides that *could not* be done, however it can be done because of the disjunction (uses *Building Or*).

1. Deduction: Goal
2. Assumption
3. Remove 2
4. Remove 3
5. Case 1:
6. Building Or 5
7. Add 6
8. Add 7
9. Case 2:
10. Add 9
11. Building Or 10
12. Add 11
13. Add 12
14. Proof By cases 5-13
15. Conclude Deduction 2-14
16. Assumption
17. Remove 16
18. Remove 17
19. Case 1:
20. Building Or 19
21. Add 20
22. Add 21
23. Case 2:
24. Building Or 23
25. Add 24
26. Add 25
27. Proof By cases 18-26
28. Conclude Deduction 15-27
29. Conclude Bi-implication 2-28

**Equivalent Statements**

**4(a)**

1. Deduction Goal:
2. Assumption
3. Quantifier Negation 2
4. Equivalence 3
5. DeMorgan’s 4
6. Deduction 2-5
7. Assumption
8. Demorgan’s 7
9. Equivalence 8
10. Quantifier Negation 9
11. Deduction 7-10
12. Conclude Deduction

**4(b)**

1. Deduction Goal:
2. Assumption
3. Assumption
4. Modus Ponens 2, 3
5. Add 4
6. Add 5
7. Conclude Deduction 4-6
8. Conclude Bi-implication 2-7
9. Bi-implication 2-8

**4(c)**

1. Deduction Goal:
2. Assumption
3. Quantifier Negation 2
4. Contrapositive 3
5. DeMorgan’s 4
6. Equivalence 5
7. Commutativity 6
8. Equivalence 7
9. Deduction 2-7
10. Assumption
11. Equivalence 10
12. Commutativity
13. DeMorgan’s
14. Contrapositive
15. Quantifier Negation
16. Deduction 10-15
17. Conclude Deduction

**4(d)**

1. Deduction Goal:
2. Assumption
3. Quantifier Negation 2
4. DeMorgan’s 3
5. DeMorgan’s 4
6. Distributive law 5
7. DeMorgan’s 6
8. Deduction 2-7
9. Assumption
10. DeMorgan’s 9
11. Distributive law 10
12. DeMorgan’s 11
13. DeMorgan’s 12
14. Quantifier Negation 13
15. Deduction 9-14
16. Conclude Deduction

**4(e)**

1. Deduction Goal:
2. Assumption
3. Equivalence 2
4. Equivalence 3
5. Equivalence 4
6. Simplification 5
7. Quantifier Negation 6
8. DeMorgan’s 7
9. Commutativity 8
10. Eval/ Build 9
11. Deduction 2-10
12. Assumption
13. DeMorgan’s 12
14. DeMorgan’s 13
15. Quantifier Negation 14
16. Remove
17. Simplification 17
18. Simplification 18
19. Implication 18, 19
20. Implication 19, 18
21. Bi-implication 20,21
22. Simplification 23
23. Simplification 24
24. A(x) Assumption
25. Eval/Build 24,26, Contradiction
26. Principle of Explosion 27
27. Implication 26-28
28. Assumption
29. Eval/Build 25, 30, Contradiction
30. Principle of Explosion 31
31. Implication 30, 32
32. Bi-implication 29, 33
33. Proof by cases 17-34
34. Add 35
35. Deduction 12-36
36. Conclude Deduction

**4(f)**

I believe it should be:

1. Deduction Goal:
2. Assumption
3. Quantifier Negation 2
4. DeMorgan’s 3
5. Quantifier Negation 4
6. Implication 2-5
7. Assumption
8. Quantifier Negation 7
9. DeMorgan’s 8
10. Quantifier Negation 9
11. Implication 7-10
12. Conclude Deduction

**Inconsistent Sets of Sentences**

**(a)** [[→x (M(x) iff [(x)) & ¬Mc] & →x ]

**(b)** [¬Fa,¬∃x (¬F(x) ∨ ¬F(x))]

**(c)** [→x →y l(x, y) → ¬∃z T(z)

(V(x))(V(y))L(x, y)(((w))C(w,w)v(3(z))T(z))

(¬→x →y L(x, y) ∨ →z B(z, z)k)

(¬(V(z))B(z, z)kv¬(3)C(w,w))

→x →y [(x, y))

**(d)** [∃x →y (H(x, y) → →w %(w,w))

(3(x))¬J(x, x)

¬(7(x))¬H(x)m)

**(e)** [→x →y (G(x, y) → Hc)

(3(x))Gi(x) &

√ x)((y))(V(z))L(x, y, z), Lcib

v¬(Hc ∨Hc)]

**(f)** [→x [(S(x) & B(x, x)) → Ka(x)]

(√x)(H(x)B(x, x))

(3(x))(S(x) & H(x)

→x ¬(Ka(x) & H(x))|