

EECS3342 Winter 2023  
Notes on Discharging POs of Refinement  
Invariant Preservation, Convergence, Deadlock Freedom  
Bridge Controller: 2nd Refinement

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# 1 Discharging the PO of Invariant Preservation: ML.out/inv2.4/INV (1st Attempt)

$d \in \mathbb{N}$   
 $d > 0$   
 $COLOUR = \{green, red\}$   
 $green \neq red$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $a \in \mathbb{N}$   
 $b \in \mathbb{N}$   
 $c \in \mathbb{N}$   
 $a + b + c = n$   
 $a = 0 \vee c = 0$   
 $ml\_hl \in COLOUR$   
 $il\_hl \in COLOUR$   
 $ml\_hl = green \Rightarrow a + b < d \wedge c = 0$   
 $il\_hl = green \Rightarrow b > 0 \wedge a = 0$   
 $ml\_hl = green$   
 $\vdash$   
 $il\_hl = green \Rightarrow b > 0 \wedge (a + 1) = 0$

**MON**

$green \neq red$   
 $il\_hl = green \Rightarrow b > 0 \wedge a = 0$   
 $ml\_hl = green$   
 $\vdash$   
 $il\_hl = green \Rightarrow b > 0 \wedge (a + 1) = 0$

**IMP\_R**

$green \neq red$   
 $il\_hl = green \Rightarrow b > 0 \wedge a = 0$   
 $ml\_hl = green$   
 $il\_hl = green$   
 $\vdash$   
 $b > 0 \wedge (a + 1) = 0$

**IMP\_L**

$green \neq red$   
 $b > 0 \wedge a = 0$   
 $ml\_hl = green$   
 $il\_hl = green$   
 $\vdash$   
 $b > 0 \wedge (a + 1) = 0$

**AND\_L**

$green \neq red$   
 $b > 0$   
 $a = 0$   
 $ml\_hl = green$   
 $il\_hl = green$   
 $\vdash$   
 $b > 0 \wedge (a + 1) = 0$

**AND\_R**

$green \neq red$   
 $b > 0$   
 $a = 0$   
 $ml\_hl = green$   
 $il\_hl = green$   
 $\vdash$   
 $b > 0$

**HYP**

$green \neq red$   
 $b > 0$   
 $a = 0$   
 $ml\_hl = green$   
 $il\_hl = green$   
 $\vdash$   
 $(a + 1) = 0$

**EQ\_LR, MON**

$green \neq red$   
 $ml\_hl = green$   
 $il\_hl = green$   
 $\vdash$   
 $(0 + 1) = 0$

**ARI**

$green \neq red$   
 $ml\_hl = green$   
 $il\_hl = green$   
 $\vdash$   
 $1 = 0$

**??**

## 2 Discharging the PO of Invariant Preservation: IL\_out/inv2.3/INV (1st Attempt)

$d \in \mathbb{N}$   
 $d > 0$   
 $COLOUR = \{green, red\}$   
 $green \neq red$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $a \in \mathbb{N}$   
 $b \in \mathbb{N}$   
 $c \in \mathbb{N}$   
 $a + b + c = n$   
 $a = 0 \vee c = 0$   
 $ml\_H \in COLOUR$   
 $il\_H \in COLOUR$   
 $ml\_H = green \Rightarrow a + b < d \wedge c = 0$   
 $il\_H = green \Rightarrow b > 0 \wedge a = 0$   
 $il\_H = green$   
 $\vdash$   
 $ml\_H = green \Rightarrow a + (b - 1) < d \wedge (c + 1) = 0$

**MON**

$green \neq red$   
 $ml\_H = green \Rightarrow a + b < d \wedge c = 0$   
 $il\_H = green$   
 $\vdash$   
 $ml\_H = green \Rightarrow a + (b - 1) < d \wedge (c + 1) = 0$

**IMP\_R**

$green \neq red$   
 $ml\_H = green \Rightarrow a + b < d \wedge c = 0$   
 $il\_H = green$   
 $ml\_H = green$   
 $\vdash$   
 $a + (b - 1) < d \wedge (c + 1) = 0$

**IMP\_L**

$green \neq red$   
 $a + b < d \wedge c = 0$   
 $il\_H = green$   
 $ml\_H = green$   
 $\vdash$   
 $a + (b - 1) < d \wedge (c + 1) = 0$

**AND\_L**

$green \neq red$   
 $a + b < d$   
 $c = 0$   
 $il\_H = green$   
 $ml\_H = green$   
 $\vdash$   
 $a + (b - 1) < d \wedge (c + 1) = 0$

**AND\_R**

$green \neq red$   
 $a + b < d$   
 $c = 0$   
 $il\_H = green$   
 $ml\_H = green$   
 $\vdash$   
 $a + (b - 1) < d$   
 $green \neq red$   
 $a + b < d$   
 $c = 0$   
 $il\_H = green$   
 $ml\_H = green$   
 $\vdash$   
 $(c + 1) = 0$

**MON**

$a + b < d$   
 $\vdash$   
 $a + (b - 1) < d$

**ARI**

**EQ\_LR, MON**

$green \neq red$   
 $il\_H = green$   
 $ml\_H = green$   
 $\vdash$   
 $(0 + 1) = 0$

**ARI**

$green \neq red$   
 $il\_H = green$   
 $ml\_H = green$   
 $\vdash$   
 $1 = 0$

**??**

### 3 Discharging the PO of Invariant Preservation: ML.out/inv2.4/INV (2nd Attempt)

$d \in \mathbb{N}$   
 $d > 0$   
 $COLOUR = \{green, red\}$   
 $green \neq red$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $a \in \mathbb{N}$   
 $b \in \mathbb{N}$   
 $c \in \mathbb{N}$   
 $a + b + c = n$   
 $a = 0 \vee c = 0$   
 $ml,hl \in COLOUR$   
 $il,hl \in COLOUR$   
 $ml,hl = green \Rightarrow a + b < d \wedge c = 0$   
 $il,hl = green \Rightarrow b > 0 \wedge a = 0$   
 $ml,hl = red \vee il,hl = red$   
 $ml,hl = green$   
 $il,hl = green \Rightarrow b > 0 \wedge (a + 1) = 0$   
 $\vdash$

MON

$green \neq red$   
 $il,hl = green \Rightarrow b > 0 \wedge a = 0$   
 $ml,hl = red \vee il,hl = red$   
 $ml,hl = green$   
 $\vdash$   
 $il,hl = green \Rightarrow b > 0 \wedge (a + 1) = 0$

IMP R

$green \neq red$   
 $il,hl = green \Rightarrow b > 0 \wedge a = 0$   
 $ml,hl = green$   
 $ml,hl = red \vee il,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $b > 0 \wedge (a + 1) = 0$

IMP L

$green \neq red$   
 $b > 0 \wedge a = 0$   
 $ml,hl = green$   
 $ml,hl = red \vee il,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $b > 0 \wedge (a + 1) = 0$

AND L

$green \neq red$   
 $b > 0$   
 $a = 0$   
 $ml,hl = green$   
 $ml,hl = red \vee il,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $b > 0 \wedge (a + 1) = 0$

AND R

$green \neq red$   
 $b > 0$   
 $a = 0$   
 $ml,hl = green$   
 $ml,hl = red \vee il,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $b > 0$

HYP

$green \neq red$   
 $b > 0$   
 $a = 0$   
 $ml,hl = green$   
 $ml,hl = red \vee il,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $(a + 1) = 0$

EQLR

MON

$green \neq red$   
 $ml,hl = green$   
 $ml,hl = red \vee il,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $(0 + 1) = 0$

ARI

$green \neq red$   
 $ml,hl = green$   
 $ml,hl = red \vee il,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $1 = 0$

OR L

$green \neq red$   
 $ml,hl = green$   
 $ml,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $1 = 0$

EQLR

MON

$green \neq red$   
 $green = red$   
 $il,hl = green$   
 $\vdash$   
 $1 = 0$

NOT L

$green = red$   
 $il,hl = green$   
 $1 \neq 0$   
 $green = red$   
 $\vdash$

HYP

$green \neq red$   
 $ml,hl = green$   
 $ml,hl = red$   
 $il,hl = green$   
 $\vdash$   
 $1 = 0$

EQLR

MON

$green \neq red$   
 $ml,hl = green$   
 $red = green$   
 $\vdash$   
 $1 = 0$

NOT L

$ml,hl = green$   
 $red = green$   
 $1 \neq 0$   
 $green = red$   
 $\vdash$

HYP

# 4 Discharging the PO of Invariant Preservation: IL.out/inv2.3/INV (2nd Attempt)

$d \in \mathbb{N}$   
 $d > 0$   
 $COLOUR = \{green, red\}$   
 $green \neq red$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $a \in \mathbb{N}$   
 $b \in \mathbb{N}$   
 $c \in \mathbb{N}$   
 $a + b + c = n$   
 $a = 0 \vee c = 0$   
 $m.HI \in COLOUR$   
 $il.HI \in COLOUR$   
 $m.HI = green \Rightarrow a + b < d \wedge c = 0$   
 $il.HI = green \Rightarrow b > 0 \wedge a = 0$   
 $m.HI = red \vee il.HI = red$   
 $il.HI = green$   
 $\vdash$   
 $m.HI = green \Rightarrow a + (b - 1) < d \wedge (c + 1) = 0$

**MON**

$green \neq red$   
 $m.HI = green \Rightarrow a + b < d \wedge c = 0$   
 $m.HI = red \vee il.HI = red$   
 $il.HI = green$   
 $\vdash$   
 $m.HI = green \Rightarrow a + (b - 1) < d \wedge (c + 1) = 0$

**IMP R**

$green \neq red$   
 $m.HI = green \Rightarrow a + b < d \wedge c = 0$   
 $il.HI = green$   
 $m.HI = red \vee il.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $a + (b - 1) < d \wedge (c + 1) = 0$

**IMP L**

$green \neq red$   
 $a + b < d \wedge c = 0$   
 $il.HI = green$   
 $m.HI = red \vee il.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $a + (b - 1) < d \wedge (c + 1) = 0$

**AND L**

$green \neq red$   
 $a + b < d$   
 $c = 0$   
 $il.HI = green$   
 $m.HI = red \vee il.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $a + (b - 1) < d \wedge (c + 1) = 0$

**AND R**

$green \neq red$   
 $a + b < d$   
 $c = 0$   
 $il.HI = green$   
 $m.HI = red \vee il.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $a + (b - 1) < d$

**MON**

$a + b < d$   
 $\vdash$   
 $a + (b - 1) < d$

**ARI**

$green \neq red$   
 $il.HI = green$   
 $m.HI = red \vee il.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $(0 + 1) = 0$

**EQ LR**

**MON**

$green \neq red$   
 $il.HI = green$   
 $m.HI = red \vee il.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $1 = 0$

**ARI**

**OR L**

$green \neq red$   
 $il.HI = green$   
 $m.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $1 = 0$

**EQ LR**

**MON**

$green \neq red$   
 $il.HI = green$   
 $il.HI = red$   
 $m.HI = green$   
 $\vdash$   
 $1 = 0$

**EQ LR**

**MON**

$green \neq red$   
 $il.HI = green$   
 $red = green$   
 $\vdash$   
 $1 = 0$

**NOT L**

$il.HI = green$   
 $red = green$   
 $1 \neq 0$   
 $\vdash$   
 $green = red$

**HYP**

$green = red$   
 $m.HI = green$   
 $1 \neq 0$   
 $\vdash$   
 $green = red$

**NOT L**

**NOT L**

**HYP**