EECS3342 Winter 2022
Notes on Discharging POs of Refinement
Invariant Preservation
File Transfer Protocol: 1st Refinement

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Contents

1 Discharing the PO of Invariant Preservation: receive/inv1_1/INV

$$\boxed{\begin{array}{l}
n > 0 \\
f \in 1\mathinner{..}n \rightarrow D \\
BOOLEAN = \{TRUE, FALSE\} \\
g \in 1\mathinner{..}n \mathrel{+\!\!\!\!\rightarrow} D \\
b \in BOOLEAN \\
b = FALSE \Rightarrow g = \varnothing \\
b = TRUE \Rightarrow g = f \\
r \in 1\mathinner{..}n+1 \\
h = (1\mathinner{..}r-1) \lhd f \\
b = TRUE \Rightarrow r = n+1 \\
r \leq n \\
\vdash \\
(r+1) \in 1\mathinner{..}n+1
\end{array}}$$

MON

$$\boxed{\begin{array}{l}
r \in 1\mathinner{..}n+1 \\
r \leq n \\
\vdash \\
(r+1) \in 1\mathinner{..}n+1
\end{array}}
\ \textbf{ARI}\
\boxed{\begin{array}{l}
1 \leq r \wedge r \leq n+1 \\
r \leq n \\
\vdash \\
\qquad 1 \leq (r+1) \\
\ \wedge \ \ (r+1) \leq n+1
\end{array}}
\ \textbf{AND\_L}\
\boxed{\begin{array}{l}
1 \leq r \\
r \leq n+1 \\
r \leq n \\
\vdash \\
\qquad 1 \leq (r+1) \\
\ \wedge \ \ (r+1) \leq n+1
\end{array}}
\ \textbf{AND\_R}\
\left\{\begin{array}{l}
\boxed{\begin{array}{l}
1 \leq r \\
r \leq n+1 \\
r \leq n \\
\vdash \\
1 \leq (r+1)
\end{array}}
\ \textbf{MON}\
\boxed{\begin{array}{l}
1 \leq r \\
\vdash \\
1 \leq (r+1)
\end{array}}
\ \textbf{ARI}
\\[2em]
\boxed{\begin{array}{l}
1 \leq r \\
r \leq n+1 \\
r \leq n \\
\vdash \\
(r+1) \leq n+1
\end{array}}
\ \textbf{MON}\
\boxed{\begin{array}{l}
r \leq n \\
\vdash \\
(r+1) \leq n+1
\end{array}}
\ \textbf{ARI}\
\boxed{\begin{array}{l}
r \leq n \\
\vdash \\
r \leq n
\end{array}}
\ \textbf{HYP}
\end{array}\right.$$

2 Discharging the PO of Invariant Preservation: receive/inv1_2/INV

$$\begin{array}{l}
n > 0 \\
f \in 1 \mathinner{..} n \to D \\
BOOLEAN = \{TRUE, FALSE\} \\
g \in 1 \mathinner{..} n \mathrel{+\!\!\!\!\rightarrow} D \\
b \in BOOLEAN \\
b = FALSE \Rightarrow g = \varnothing \\
b = TRUE \Rightarrow g = f \\
r \in 1 \mathinner{..} n + 1 \\
h = (1 \mathinner{..} r - 1) \lhd f \\
b = TRUE \Rightarrow r = n + 1 \\
r \leq n \\
\vdash \\
h \cup \{(r, f(r))\} = (1 \mathinner{..} (r + 1) - 1) \lhd f
\end{array}$$

MON

$$\begin{array}{l}
f \in 1 \mathinner{..} n \to D \\
r \in 1 \mathinner{..} n + 1 \\
h = (1 \mathinner{..} r - 1) \lhd f \\
r \leq n \\
\vdash \\
h \cup \{(r, f(r))\} = (1 \mathinner{..} (r + 1) - 1) \lhd f
\end{array}$$

ARI

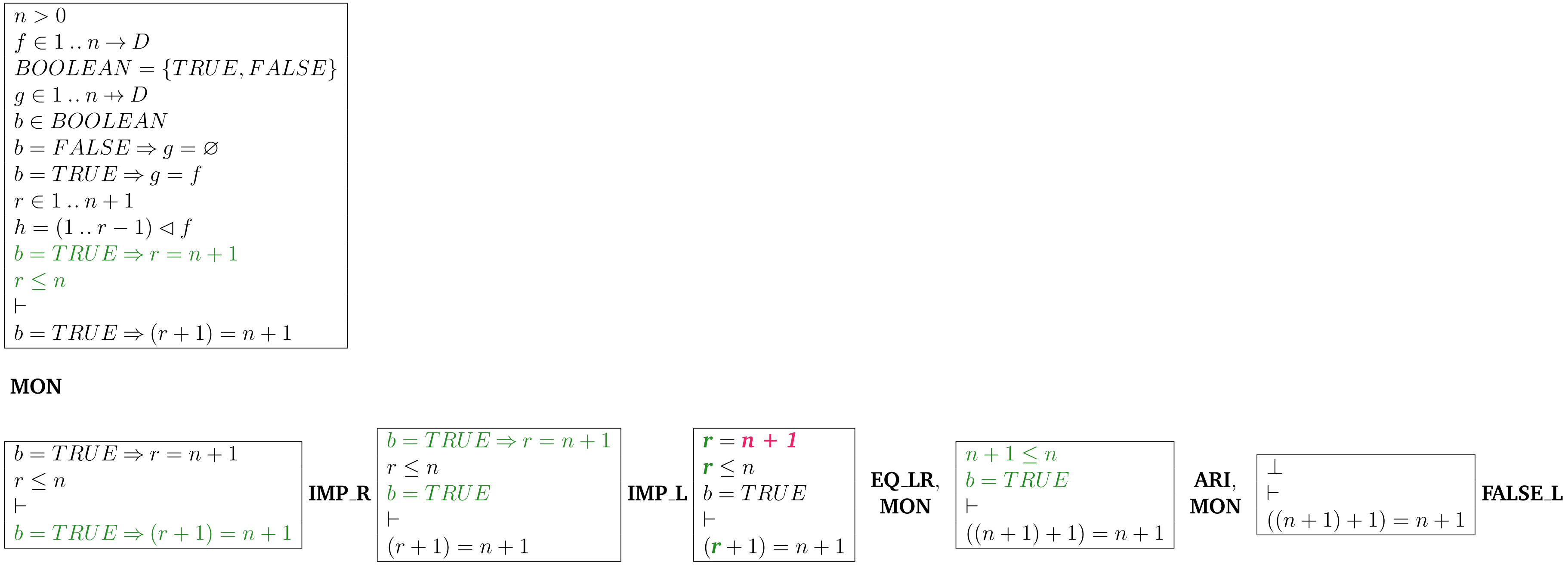
$$\begin{array}{l}
f \in 1 \mathinner{..} n \to D \\
1 \leq r \\
h = (1 \mathinner{..} r - 1) \lhd f \\
r \leq n \\
\vdash \\
h \cup \{(r, f(r))\} = (1 \mathinner{..} (r + 1) - 1) \lhd f
\end{array}$$

EQ_LR, MON, ARI

$$\begin{array}{l}
f \in 1 \mathinner{..} n \to D \\
1 \leq r \\
r \leq n \\
\vdash \\
(1 \mathinner{..} r - 1) \lhd f \cup \{(r, f(r))\} = (1 \mathinner{..} r) \lhd f
\end{array}$$

ARI

3 Discharging the PO of Invariant Preservation: receive/inv1_3/INV



$n > 0$
$f \in 1 \mathinner{..} n \rightarrow D$
$BOOLEAN = \{TRUE, FALSE\}$
$g \in 1 \mathinner{..} n \nrightarrow D$
$b \in BOOLEAN$
$b = FALSE \Rightarrow g = \varnothing$
$b = TRUE \Rightarrow g = f$
$r \in 1 \mathinner{..} n + 1$
$h = (1 \mathinner{..} r - 1) \lhd f$
$b = TRUE \Rightarrow r = n + 1$
$r \leq n$
$\vdash$
$b = TRUE \Rightarrow (r + 1) = n + 1$

MON

$b = TRUE \Rightarrow r = n + 1$
$r \leq n$
$\vdash$
$b = TRUE \Rightarrow (r + 1) = n + 1$

IMP_R

$b = TRUE \Rightarrow r = n + 1$
$r \leq n$
$b = TRUE$
$\vdash$
$(r + 1) = n + 1$

IMP_L

$\boldsymbol{r} = \boldsymbol{n + 1}$
$\boldsymbol{r} \leq n$
$b = TRUE$
$\vdash$
$(\boldsymbol{r} + 1) = n + 1$

EQ_LR, MON

$n + 1 \leq n$
$b = TRUE$
$\vdash$
$((n + 1) + 1) = n + 1$

ARI, MON

$\bot$
$\vdash$
$((n + 1) + 1) = n + 1$

FALSE_L