EECS2011-N&Z Winter 2022 **Name (Print):** _____
Fundamentals of Data Structures
Example Exam Questions
April 26, 2022 PPY Login _____
Time Limit: 180 Minutes Signature _____

This exam contains 6 pages (including this cover page) and 3 problems.

Check to see if any pages are missing.

Do not detach any question pages from the booklet.

Enter **all** requested information on the top of this page before you start the exam, and put your **initials** on the top of every page, in case the pages become separated.

Attempt **all** questions. Answer each question in the boxed space provided.

The following rules apply:

- **NO QUESTIONS DURING THE EXAM.**
- **If a question is ambiguous or unclear, then please write your assumptions and proceed to answer the question.**
- All answers must appear in the boxed areas in this booklet.
- Only writings within the designated answer boxes will be graded. Plan your answers on the sketch paper provided.
- **Write in valid Java syntax** wherever required.
- Where descriptive answers are requested, use complete sentences and paragraphs. Be precise and concise.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive credit**. A correct answer, unsupported by calculations or explanation will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in this table which contains your raw mark scores.

Problem	Points	Score
1	20	
2	30	
3	50	
Total:	100	

1. Consider the following fragment of Java code:

```
1  boolean containsDuplicate (int[] a, int n) {
2    for (int i = 0; i < n; ) {
3      for (int j = 0; j < n; ) {
4        if (i != j && a[i] == a[j]) {
5          return true; }
6        j ++; }
7      i ++; }
8    return false; }
```

Derive, in the worst case, the number of primitive operations executed to return the result.

Solution:

- The answer is: $9n^2 + 5n + 3$
- See the solution walkthrough here: https://www.youtube.com/watch?v=k-ijBQgmBtY&list=PL5dxAmCmjv_6EOKnlgJJ4OEKC7ZqJ0Hsv&index=4&t=796s
- Solution notes here: https://www.eecs.yorku.ca/~jackie/teaching/lectures/2022/W/EECS2011/blackboards/Blackboard-EECS2011-W22-Q&A-20220120.pdf#page=3

[of 20 marks]

2. (a) Given a BST rooted at node n, describe how an entry (k, v) can be inserted.

Requirements. Do not write any Java code. Describe the steps precisely and concisely.

Solution:

- Let node **p** be the return value from `search(n, k)`.
- If **p** is an **internal node**
  ⇒ Key k exists in the BST.
  ⇒ Set p's value to v.
- If **p** is an **external node**
  ⇒ Key k deos **not** exist in the BST.
  ⇒ Set p's key and value to k and v.

[of 20 marks]

(b) Explain why an *inorder* traversal of a binary search tree produces a sequence of entries whose keys are sorted in an *increasing* order.

Solution:

- A binary search tree must satisfy the search property: all nodes on the LST of the root have keys that are strictly less than that of the root; and all nodes on the RSH of the root have keys that are strictly less than that of the root.
- For each subtree, say rooted at r, in a binary search tree, an inorder traversal first visits r's LST, then r, and then r's RST.
- Consequently, the produced sequence of keys are sorted in an increasing order.

[of 10 marks]

3. (a) Consider the following classes of functions:
   - $O(n)$
   - $O(log(n))$
   - $O(n^2)$
   - $O(1)$
   - $O(2^n)$
   - $O(n^3)$
   - $O(n \cdot log(n))$

   Say each of the above functions maps from input size n to the *approximated* algorithm running time. Sort, from left to right, the above classes of functions from the cheapest to the most expensive. **Caution:** You will lose **all** marks if the order is not completely correct.

   Solution: $O(1)$ $O(log(n))$ $O(n)$ $O(n \cdot log(n))$ $O(n^2)$ $O(n^3)$ $O(2^n)$

   [of 10 marks]

   For Part (b) to Part (d), consider the following statements:

   (**A**) $3n + 7$ is $O(n \cdot log(n))$
   (**B**) $3n + 7$ is $O(n)$
   (**C**) $3n + 7$ is $O(1)$
   (**D**) $3n + 7$ is $O(2^n)$
   (**E**) $3n + 7$ is $O(log(n))$
   (**F**) $3n + 7$ is $O(n^2)$

   (b) Which of the above statement or statements are *correct*? Do not guess: you lose **all** marks if you make a mistake.

   Solution: Statements **A** **B** **D** **F**

   [of 10 marks]

   (c) Among the above statement or statements that are *correct*, which **one** is the most *accurate*?

   Solution: Statement **B**

   [of 10 marks]

   (d) Justify your answer to the previous question. That is, clearly explain why it is more *accurate* than all other *correct* statements.

   Solution: The highest power of n in $3n + 7$ is one. So Statement B is the most accurate by saying that $3n+7$ is $O(n)$. The class $O(n)$ is strictly contained by $O(n \cdot log(n))$, which is strictly contained by $O(n^2)$, which is strictly contained by $O(2^n)$.

   [of 10 marks]

   (e) Prove that $f(n) = 4n^3 - 5n^2 + 59 + n^4 + 9n$ is $O(n^4)$.

   Solution: Choose $c = 78$ and $n_0 = 1$.

   [of 10 marks]

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