

This exam contains 6 pages (including this cover page) and 3 problems.

Check to see if any pages are missing.

Do not detach any question pages from the booklet.

Enter **all** requested information on the top of this page before you start the exam, and put your **initials** on the top of every page, in case the pages become separated.

Attempt **all** questions. Answer each question in the boxed space provided.

The following rules apply:

- **NO QUESTIONS DURING THE EXAM.**
- **If a question is ambiguous or unclear, then please write your assumptions and proceed to answer the question.**
- All answers must appear in the boxed areas in this booklet.
- Only writings within the designated answer boxes will be graded. Plan your answers on the sketch paper provided.
- **Write in valid Java syntax** wherever required.
- Where descriptive answers are requested, use complete sentences and paragraphs. Be precise and concise.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive credit.** A correct answer, unsupported by calculations or explanation will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in this table which contains your raw mark scores.

Problem	Points	Score
1	20	
2	30	
3	50	
Total:	100	

1. Consider the following fragment of Java code:

```
1 boolean containsDuplicate (int[] a, int n) {  
2   for (int i = 0; i < n; ) {  
3     for (int j = 0; j < n; ) {  
4       if (i != j && a[i] == a[j]) {  
5         return true; }  
6       j ++; }  
7     i ++; }  
8   return false; }
```

Derive, in the worst case, the number of primitive operations executed to return the result.

2. (a) Given a BST rooted at node n , describe how an entry (k, v) can be inserted.

Requirements. Do not write any Java code. Describe the steps precisely and concisely.

[of 20 marks]

- (b) Explain why an *inorder* traversal of a binary search tree produces a sequence of entries whose keys are sorted in an *increasing* order.

[of 10 marks]

3. (a) Consider the following classes of functions:

- $O(n)$
- $O(\log(n))$
- $O(n^2)$
- $O(1)$
- $O(2^n)$
- $O(n^3)$
- $O(n \cdot \log(n))$

Say each of the above functions maps from input size n to the *approximated* algorithm running time. Sort, from left to right, the above classes of functions from the cheapest to the most expensive.

Caution: You will lose **all** marks if the order is not completely correct.

[of 10 marks]

For Part (b) to Part (d), consider the following statements:

- (A) $3n + 7$ is $O(n \cdot \log(n))$
- (B) $3n + 7$ is $O(n)$
- (C) $3n + 7$ is $O(1)$
- (D) $3n + 7$ is $O(2^n)$
- (E) $3n + 7$ is $O(\log(n))$
- (F) $3n + 7$ is $O(n^2)$

(b) Which of the above statement or statements are *correct*? Do not guess: you lose **all** marks if you make a mistake.

[of 10 marks]

(c) Among the above statement or statements that are *correct*, which **one** is the most *accurate*?

[of 10 marks]

(d) Justify your answer to the previous question. That is, clearly explain why it is more *accurate* than all other *correct* statements.

[of 10 marks]

(e) Prove that $f(n) = 4n^3 - 5n^2 + 59 + n^4 + 9n$ is $O(n^4)$.

[of 10 marks]

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