Parser: Syntactic Analysis

Readings: EAC2 Chapter 3

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Compilers and Interpreters
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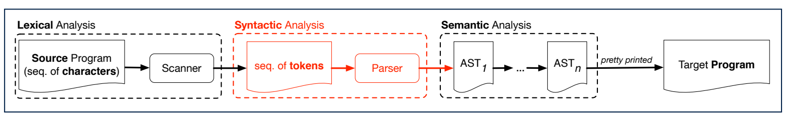
Parser in Context

- Recall:



- Treats the input programas as a ***a sequence of classified tokens/words***
- Applies rules **parsing** token sequences as ***abstract syntax trees (ASTs)*** [***syntactic*** analysis]
- Upon termination:
  - Reports token sequences not derivable as ASTs
  - Produces an ***AST***
- No longer considers ***every character*** in input program.
- *Derivable* token sequences constitute a *context-free language (CFL)*.

Context-Free Languages: Introduction

- We have seen ***regular languages***:
  - Can be described using ***finite automata*** or ***regular expressions***.
  - Satisfy the ***pumping lemma***.
- Language with ***recursive*** structures are provably ***non-regular***. e.g., $\{0^n1^n \mid n \geq 0\}$
- *Context-Free Grammars (CFG's)* are used to describe strings that can be generated in a ***recursive*** fashion.
- *Context-Free Languages (CFL's)* are:
  - Languages that can be described using CFG's.
  - A proper superset of the set of regular languages.

CFG: Example (1.1)

- The following language that is ***non-regular***

$$\{0^n\#1^n \mid n \geq 0\}$$

can be described using a ***context-free grammar (CFG)***:

$$\begin{array}{lcl} A & \rightarrow & 0A1 \\ A & \rightarrow & B \\ B & \rightarrow & \# \end{array}$$

- A grammar contains a collection of ***substitution*** or ***production*** rules, where:
  - A **terminal** is a word $w \in \Sigma^*$ (e.g., 0, 1, *etc.*).
  - A ***variable*** or ***non-terminal*** is a word $w \notin \Sigma^*$ (e.g., A, B, *etc.*).
  - A ***start variable*** occurs on the LHS of the topmost rule (e.g., A).

CFG: Example (1.2)

- Given a grammar, generate a string by:
  1. Write down the ***start variable***.
  2. Choose a production rule where the ***start variable*** appears on the LHS of the arrow, and ***substitute*** it by the RHS.
  3. There are two cases of the re-written string:
     - 3.1 It contains **no** variables, then you are done.
     - 3.2 It contains **some** variables, then ***substitute*** each variable using the relevant ***production rules***.
  4. Repeat Step 3.
- e.g., We can generate an infinite number of strings from

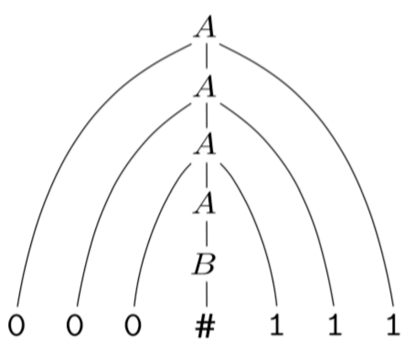
$$\begin{array}{lcl} A & \rightarrow & 0A1 \\ A & \rightarrow & B \\ B & \rightarrow & \# \end{array}$$

  - $A \Rightarrow B \Rightarrow \#$
  - $A \Rightarrow 0A1 \Rightarrow 0B1 \Rightarrow 0\#1$
  - $A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 00B11 \Rightarrow 00\#11$
  - ...

CFG: Example (1.2)

Given a CFG, a string's ***derivation*** can be shown as a ***parse tree***.

e.g., The derivation of $000\#111$ has the parse tree



CFG: Example (2)

Design a CFG for the following language:

$$\{w \mid w \in \{0,1\}^* \land w \texttt{ is a palidrome}\}$$

e.g., 00, 11, 0110, 1001, *etc.*

$$\begin{array}{lcl} P & \rightarrow & \epsilon \\ P & \rightarrow & 0 \\ P & \rightarrow & 1 \\ P & \rightarrow & 0P0 \\ P & \rightarrow & 1P1 \end{array}$$

CFG: Example (3)

Design a CFG for the following language:

$$\{ww^R \mid w \in \{0,1\}^*\}$$

e.g., 00, 11, 0110, *etc.*

$$\begin{array}{lcl} P & \rightarrow & \epsilon \\ P & \rightarrow & 0P0 \\ P & \rightarrow & 1P1 \end{array}$$

CFG: Example (4)

Design a CFG for the set of binary strings, where each block of 0's followed by at least as many 1's.

e.g., 000111, 0001111, *etc.*

- We use S to represent one such string, and A to represent each such block in S.

$$
\begin{array}{llll}
S & \rightarrow & \epsilon & \{\texttt{BC of } S\} \\
S & \rightarrow & AS & \{\texttt{RC of } S\} \\
A & \rightarrow & \epsilon & \{\texttt{BC of } A\} \\
A & \rightarrow & 01 & \{\texttt{BC of } A\} \\
A & \rightarrow & 0A1 & \{\texttt{RC of } A\texttt{: equal 0's and 1's}\} \\
A & \rightarrow & A1 & \{\texttt{RC of } A\texttt{: more 1's}\}
\end{array}
$$

CFG: Example (5.1) Version 1

Design the grammar for the following small expression language, which supports:

- Arithmetic operations: +, −, *, /
- Relational operations: >, <, >=, <=, ==, /=
- Logical operations: `true`, `false`, `!`, `&&`, `||`, `=>`

  Start with the variable ***Expression***.
- There are two possible versions:
  1. All operations are <u>mixed</u> together.
  2. Relevant operations are <u>grouped</u> together.
     Try both!

CFG: Example (5.2) Version 1

$$
\begin{array}{lll}
\mathit{Expression} & \rightarrow & \mathit{IntegerConstant} \\
& | & -\mathit{IntegerConstant} \\
& | & \mathit{BooleanConstant} \\
& | & \mathit{BinaryOp} \\
& | & \mathit{UnaryOp} \\
& | & (\ \mathit{Expression}\ ) \\
\\
\mathit{IntegerConstant} & \rightarrow & \mathit{Digit} \\
& | & \mathit{Digit}\ \mathit{IntegerConstant} \\
\\
\mathit{Digit} & \rightarrow & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\
\\
\mathit{BooleanConstant} & \rightarrow & \texttt{TRUE} \\
& | & \texttt{FALSE}
\end{array}
$$

CFG: Example (5.3) Version 1

$$
\begin{array}{lll}
\mathit{BinaryOp} & \rightarrow & \mathit{Expression} + \mathit{Expression} \\
& | & \mathit{Expression} - \mathit{Expression} \\
& | & \mathit{Expression} * \mathit{Expression} \\
& | & \mathit{Expression}\ /\ \mathit{Expression} \\
& | & \mathit{Expression}\ \&\&\ \mathit{Expression} \\
& | & \mathit{Expression}\ ||\ \mathit{Expression} \\
& | & \mathit{Expression} => \mathit{Expression} \\
& | & \mathit{Expression} == \mathit{Expression} \\
& | & \mathit{Expression}\ /= \mathit{Expression} \\
& | & \mathit{Expression} > \mathit{Expression} \\
& | & \mathit{Expression} < \mathit{Expression} \\
\\
\mathit{UnaryOp} & \rightarrow & !\ \mathit{Expression}
\end{array}
$$

CFG: Example (5.4) Version 1

However, Version 1 of CFG:

- ***Parses*** string that requires further ***semantic analysis*** (e.g., type checking):
  e.g., `3 => 6`
- Is ***ambiguous***, meaning?
  - Some string may have more than one ways to interpreting it.
  - An interpretation is either visualized as a ***parse tree***, or written as a sequence of ***derivations***.

  e.g., Draw the parse tree(s) for `3 * 5 + 4`

CFG: Example (5.5) Version 2

$$
\begin{array}{lcl}
\mathit{Expression} & \rightarrow & \mathit{ArithmeticOp} \\
 & \mid & \mathit{RelationalOp} \\
 & \mid & \mathit{LogicalOp} \\
 & \mid & \texttt{(}\ \mathit{Expression}\ \texttt{)} \\
\\
\mathit{IntegerConstant} & \rightarrow & \mathit{Digit} \\
 & \mid & \mathit{Digit}\ \mathit{IntegerConstant} \\
\\
\mathit{Digit} & \rightarrow & 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\
\\
\mathit{BooleanConstant} & \rightarrow & \texttt{TRUE} \\
 & \mid & \texttt{FALSE}
\end{array}
$$

CFG: Example (5.6) Version 2

$$
\begin{array}{lcl}
\mathit{ArithmeticOp} & \rightarrow & \mathit{ArithmeticOp}\ \texttt{+}\ \mathit{ArithmeticOp} \\
 & \mid & \mathit{ArithmeticOp}\ \texttt{-}\ \mathit{ArithmeticOp} \\
 & \mid & \mathit{ArithmeticOp}\ \texttt{*}\ \mathit{ArithmeticOp} \\
 & \mid & \mathit{ArithmeticOp}\ \texttt{/}\ \mathit{ArithmeticOp} \\
 & \mid & \texttt{(}\mathit{ArithmeticOp}\texttt{)} \\
 & \mid & \mathit{IntegerConstant} \\
 & \mid & \texttt{-}\mathit{IntegerConstant} \\
\mathit{RelationalOp} & \rightarrow & \mathit{ArithmeticOp}\ \texttt{==}\ \mathit{ArithmeticOp} \\
 & \mid & \mathit{ArithmeticOp}\ \texttt{/=}\ \mathit{ArithmeticOp} \\
 & \mid & \mathit{ArithmeticOp}\ \texttt{>}\ \mathit{ArithmeticOp} \\
 & \mid & \mathit{ArithmeticOp}\ \texttt{<}\ \mathit{ArithmeticOp} \\
\mathit{LogicalOp} & \rightarrow & \mathit{LogicalOp}\ \texttt{\&\&}\ \mathit{LogicalOp} \\
 & \mid & \mathit{LogicalOp}\ \texttt{||}\ \mathit{LogicalOp} \\
 & \mid & \mathit{LogicalOp}\ \texttt{=>}\ \mathit{LogicalOp} \\
 & \mid & \texttt{!}\ \mathit{LogicalOp} \\
 & \mid & \texttt{(}\mathit{LogicalOp}\texttt{)} \\
 & \mid & \mathit{RelationalOp} \\
 & \mid & \mathit{BooleanConstant}
\end{array}
$$

CFG: Example (5.7) Version 2

However, Version 2 of CFG:

- Eliminates some cases for further semantic analysis:
  e.g., `(1 + 2) => (5 / 4)` [no parse tree]
- Still ***parses*** strings that might require further ***semantic analysis***:
  e.g., `(1 + 2) / (5 - (2 + 3))`
- Still is ***ambiguous***.
  e.g., Draw the parse tree(s) for `3 * 5 + 4`

CFG: Formal Definition (1)

- A ***context-free grammar (CFG)*** is a 4-tuple (V, Σ, R, S):
  - V is a finite set of **variables**.
  - Σ is a finite set of ***terminals***. $[V \cap \Sigma = \varnothing]$
  - R is a finite set of ***rules*** s.t.

$$R \subseteq \{v \rightarrow s \mid v \in V \land s \in (V \cup \Sigma)^*\}$$

  - $S \in V$ is is the **start variable**.
- Given strings $u, v, w \in (V \cup \Sigma)^*$, variable $A \in V$, a rule $A \rightarrow w$:
  - $\boxed{uAv \Rightarrow uwv}$ menas that uAv ***yields*** uwv.
  - $\boxed{u \overset{*}{\Rightarrow} v}$ means that u ***derives*** v, if:
    - $u = v$; or
    - $u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v$ [a ***yield sequence***]
- Given a CFG $G = (V, \Sigma, R, S)$, the language of G

$$L(G) = \{w \in \Sigma^* \mid S \overset{*}{\Rightarrow} w\}$$

CFG: Formal Definition (2): Example

- Design the ***CFG*** for strings of properly-nested parentheses.
  e.g., $()$, $()()$, $((()()))()$, *etc.*
  Present your answer in a ***formal*** manner.
- $G = (\{S\}, \{(,)\}, R, S)$, where R is

$$S \rightarrow (\ S\ ) \mid SS \mid \epsilon$$

- Draw ***parse trees*** for the above three strings that G generates.

CFG: Formal Definition (3): Example

- Consider the grammar $G = (V, \Sigma, R, S)$:
  - R is

$$\begin{array}{lcl} Expr & \rightarrow & Expr + Term \\ & \mid & Term \\ Term & \rightarrow & Term * Factor \\ & \mid & Factor \\ Factor & \rightarrow & (Expr) \\ & \mid & \mathtt{a} \end{array}$$

  - $V = \{Expr, Term, Factor\}$
  - $\Sigma = \{\mathtt{a}, +, *, (,)\}$
  - $S = Expr$
- ***Precedence*** of operators $+$, $*$ is embedded in the grammar.
  - "Plus" is specified at a **higher** level (*Expr*) than is "times" (*Term*).
  - Both operands of a multiplication (*Factor*) may be **parenthesized**.

Regular Expressions to CFG's

- Recall the semantics of regular expressions (assuming that we do not consider $\varnothing$):

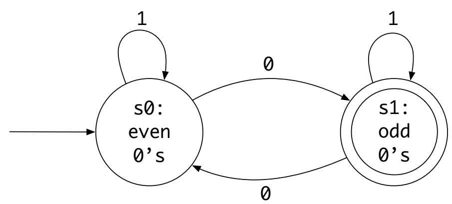
$$\begin{array}{lcl} L(\ \epsilon\ ) & = & \{\epsilon\} \\ L(\ a\ ) & = & \{a\} \\ L(\ E + F\ ) & = & L(E) \cup L(F) \\ L(\ EF\ ) & = & L(E)L(F) \\ L(\ E^*\ ) & = & (L(E))^* \\ L(\ (E)\ ) & = & L(E) \end{array}$$

- e.g., Grammar for $(00 + 1)^* + (11 + 0)^*$

$$\begin{array}{lcl} S & \rightarrow & A \mid B \\ A & \rightarrow & \epsilon \mid AC \\ C & \rightarrow & 00 \mid 1 \\ B & \rightarrow & \epsilon \mid BD \\ D & \rightarrow & 11 \mid 0 \end{array}$$

DFA to CFG's

- Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$:
  - Make a ***variable*** R_i for each ***state*** $q_i \in Q$.
  - Make R_0 the ***start variable***, where q_0 is the ***start state*** of M.
  - Add a rule $R_i \rightarrow aR_j$ to the grammar if $\delta(q_i, a) = q_j$.
  - Add a rule $R_i \rightarrow \epsilon$ if $q_i \in F$.
- e.g., Grammar for



$$
\begin{array}{lcl}
R_0 & \rightarrow & 1R_0 \mid 0R_1 \\
R_1 & \rightarrow & 0R_0 \mid 1R_1 \mid \epsilon
\end{array}
$$

CFG: Leftmost Derivations (1)

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr + Term \mid Term \\
Term & \rightarrow & Term * Factor \mid Factor \\
Factor & \rightarrow & (Expr) \mid a
\end{array}
$$

- Given a string ($\in (V \cup \Sigma)^*$), a ***left-most derivation (LMD)*** keeps substituting the leftmost non-terminal ($\in V$).
- ***Unique LMD*** for the string `a + a * a`:

$$
\begin{array}{ll}
Expr & \Rightarrow Expr + Term \\
 & \Rightarrow Term + Term \\
 & \Rightarrow Factor + Term \\
 & \Rightarrow a + Term \\
 & \Rightarrow a + Term * Factor \\
 & \Rightarrow a + Factor * Factor \\
 & \Rightarrow a + a * Factor \\
 & \Rightarrow a + a * a
\end{array}
$$

- This ***LMD*** suggests that `a * a` is the right operand of `+`.

CFG: Rightmost Derivations (1)

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr + Term \mid Term \\
Term & \rightarrow & Term * Factor \mid Factor \\
Factor & \rightarrow & (Expr) \mid a
\end{array}
$$

- Given a string ($\in (V \cup \Sigma)^*$), a ***right-most derivation (RMD)*** keeps substituting the rightmost non-terminal ($\in V$).
- ***Unique RMD*** for the string `a + a * a`:

$$
\begin{array}{ll}
Expr & \Rightarrow Expr + Term \\
 & \Rightarrow Expr + Term * Factor \\
 & \Rightarrow Expr + Term * a \\
 & \Rightarrow Expr + Factor * a \\
 & \Rightarrow Expr + a * a \\
 & \Rightarrow Term + a * a \\
 & \Rightarrow Factor + a * a \\
 & \Rightarrow a + a * a
\end{array}
$$

- This ***RMD*** suggests that `a * a` is the right operand of `+`.

CFG: Leftmost Derivations (2)

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr + Term \mid Term \\
Term & \rightarrow & Term * Factor \mid Factor \\
Factor & \rightarrow & (Expr) \mid a
\end{array}
$$

- ***Unique LMD*** for the string `(a + a) * a`:

$$
\begin{array}{ll}
Expr & \Rightarrow Term \\
 & \Rightarrow Term * Factor \\
 & \Rightarrow Factor * Factor \\
 & \Rightarrow (Expr) * Factor \\
 & \Rightarrow (Expr + Term) * Factor \\
 & \Rightarrow (Term + Term) * Factor \\
 & \Rightarrow (Factor + Term) * Factor \\
 & \Rightarrow (a + Term) * Factor \\
 & \Rightarrow (a + Factor) * Factor \\
 & \Rightarrow (a + a) * Factor \\
 & \Rightarrow (a + a) * a
\end{array}
$$

- This ***LMD*** suggests that `(a + a)` is the left operand of `*`.

CFG: Rightmost Derivations (2)

$$
\begin{array}{lcl}
\textit{Expr} & \rightarrow & \textit{Expr} \ + \ \textit{Term} \mid \textit{Term} \\
\textit{Term} & \rightarrow & \textit{Term} \ * \ \textit{Factor} \mid \textit{Factor} \\
\textit{Factor} & \rightarrow & (\ \textit{Expr}\ ) \mid a
\end{array}
$$

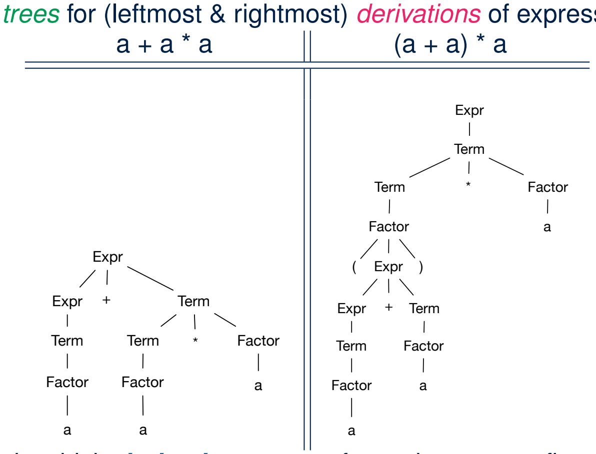
- ***Unique RMD*** for the string `(a + a) * a`:

$$
\begin{array}{lcl}
\textit{Expr} & \Rightarrow & \textit{Term} \\
 & \Rightarrow & \textit{Term} \ * \ \textit{Factor} \\
 & \Rightarrow & \textit{Term} \ * \ a \\
 & \Rightarrow & \textit{Factor} \ * \ a \\
 & \Rightarrow & (\ \textit{Expr}\ ) \ * \ a \\
 & \Rightarrow & (\ \textit{Expr} + \textit{Term}\ ) \ * \ a \\
 & \Rightarrow & (\ \textit{Expr} + \textit{Factor}\ ) \ * \ a \\
 & \Rightarrow & (\ \textit{Expr} + a\ ) \ * \ a \\
 & \Rightarrow & (\ \textit{Term} + a\ ) \ * \ a \\
 & \Rightarrow & (\ \textit{Factor} + a\ ) \ * \ a \\
 & \Rightarrow & (\ a + a\ ) \ * \ a
\end{array}
$$

- This ***RMD*** suggests that `(a + a)` is the left operand of `*`.

CFG: Parse Trees vs. Derivations (1)

- *Parse trees* for (leftmost & rightmost) *derivations* of expressions:



- Orders in which ***derivations*** are performed are ***not*** reflected on parse trees.

CFG: Parse Trees vs. Derivations (2)

- A string $w \in \Sigma^*$ may have <u>more than one</u> ***derivations***.

  Q: distinct ***derivations*** for $w \in \Sigma^* \Rightarrow$ distinct ***parse trees*** for w?

  A: Not in general $\because$ Derivations with ***distinct orders*** of variable substitutions may still result in the ***same parse tree***.
- For example:

$$
\begin{array}{lcl}
\textit{Expr} & \rightarrow & \textit{Expr} \ + \ \textit{Term} \mid \textit{Term} \\
\textit{Term} & \rightarrow & \textit{Term} \ * \ \textit{Factor} \mid \textit{Factor} \\
\textit{Factor} & \rightarrow & (\ \textit{Expr}\ ) \mid a
\end{array}
$$

For string `a + a * a`, the ***LMD*** and ***RMD*** have ***distinct orders*** of variable substitutions, but their corresponding ***parse trees are the <u>same</u>***.

CFG: Ambiguity: Definition

Given a grammar $G = (V, \Sigma, R, S)$:

- A string $w \in \Sigma^*$ is derived *ambiguously* in G if there exist two or more ***distinct parse trees*** or, equally, two or more ***distinct LMDs*** or, equally, two or more ***distinct RMDs***.

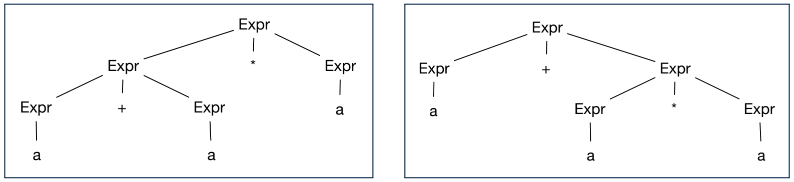
  We require that all such derivations are completed by following a <u>consisten</u> order (**leftmost** or **rightmost**) to avoid ***false positive***.
- G is *ambiguous* if it generates some string ambiguously.

CFG: Ambiguity: Exercise (1)

- Is the following grammar *ambiguous*?

$$Expr \rightarrow Expr\ \texttt{+}\ Expr \mid Expr\ \texttt{*}\ Expr \mid \texttt{(}\ Expr\ \texttt{)} \mid a$$

- Yes $\because$ it generates the string `a + a * a` *ambiguously*:



- ***Distinct ASTs*** (for the ***same input***) imply ***distinct semantic interpretations***: e.g., a pre-order traversal for evaluation
- **Exercise**: Show ***LMDs*** for the two parse trees.

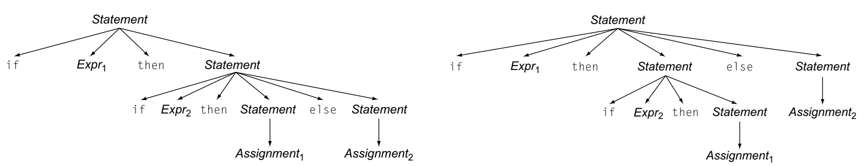
CFG: Ambiguity: Exercise (2.1)

- Is the following grammar *ambiguous*?

$$\begin{array}{lcl} Statement & \rightarrow & \texttt{if}\ Expr\ \texttt{then}\ Statement \\ & \mid & \texttt{if}\ Expr\ \texttt{then}\ Statement\ \texttt{else}\ Statement \\ & \mid & Assignment \\ & \ldots & \end{array}$$

- Yes $\because$ it derives the following string *ambiguously*:

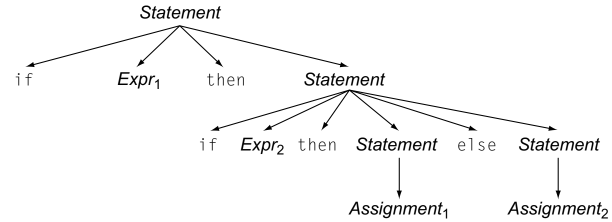
$\texttt{if}\ Expr_1\ \texttt{then}\ \texttt{if}\ Expr_2\ \texttt{then}\ Assignment_1\ \texttt{else}\ Assignment_2$



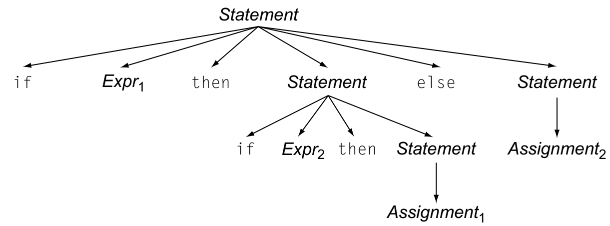
- This is called the ***dangling else*** problem.
- **Exercise:** Show ***LMDs*** for the two parse trees.

CFG: Ambiguity: Exercise (2.2)

(***Meaning 1***) $Assignment_2$ may be associated with the inner `if`:



(***Meaning 2***) $Assignment_2$ may be associated with the outer `if`:



CFG: Ambiguity: Exercise (2.3)

- We may remove the ***ambiguity*** by specifying that the *dangling* `else` is associated with the **nearest** `if`:

$$\begin{array}{lcl} Statement & \rightarrow & \texttt{if}\ Expr\ \texttt{then}\ Statement \\ & \mid & \texttt{if}\ Expr\ \texttt{then}\ WithElse\ \texttt{else}\ Statement \\ & \mid & Assignment \\ WithElse & \rightarrow & \texttt{if}\ Expr\ \texttt{then}\ WithElse\ \texttt{else}\ WithElse \\ & \mid & Assignment \end{array}$$

- When applying $\boxed{\texttt{if}\ \ldots\ \texttt{then}\ WithElse\ \texttt{else}\ Statement}$:
  - The ***true*** branch will be produced via *WithElse*.
  - The ***false*** branch will be produced via *Statement*.

  There is ***no circularity*** between the two non-terminals.

Discovering Derivations

- Given a CFG $G = (V, \Sigma, R, S)$ and an input program $p \in \Sigma^*$:
  - So far we **manually** come up a valid ***derivation*** s.t. $S \stackrel{*}{\Rightarrow} p$.
  - A *parser* is supposed to **automate** this ***derivation*** process.
    - Input: **A sequence of (t, c) pairs**, where each ***token*** t (e.g., `r241`) belongs to a ***syntactic category*** c (e.g., `register`); and a ***CFG*** G.
    - Output: A ***valid derivation*** (as an ***AST***); or A ***parse error***.
- In the process of constructing an ***AST*** for the input program:
  - ***Root*** of AST: The ***start symbol*** S of G
  - ***Internal nodes***: A ***subset of variables*** V of G
  - ***Leaves*** of AST: A ***token/terminal*** sequence

  $\Rightarrow$ Discovering the *grammatical connections* (w.r.t. R of G) between the ***root***, ***internal nodes***, and ***leaves*** is the hard part!
- Approaches to Parsing: $[\, w \in (V \cup \Sigma)^*,\ A \in V,\ \boxed{A \rightarrow w} \in R \,]$
  - **Top-down** parsing

    For a node representing A, extend it with a subtree representing w.
  - **Bottom-up** parsing

    For a substring matching w, build a node representing A accordingly.

TDP: Discovering Leftmost Derivation

```
ALGORITHM: TDParse
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: Root of a Parse Tree or Syntax Error
PROCEDURE:
  root := a new node for the start symbol S
  focus := root
  initialize an empty stack trace
  trace.push(null)
  word := NextWord()
  while (true):
    if focus ∈ V then
      if ∃ unvisited rule focus → β1 β2 ... βn ∈ R then
        create β1, β2 ... βn as children of focus
        trace.push(βn βn−1 ... β2)
        focus := β1
      else
        if focus = S then report syntax error
        else backtrack
    elseif word matches focus then
      word := NextWord()
      focus := trace.pop()
    elseif word = EOF ∧ focus = null then return root
    else backtrack
```

backtrack ≜ pop *focus*.siblings; *focus* := *focus*.parent; *focus*.resetChildren

TDP: Exercise (1)

- Given the following CFG **G**:

$$
\begin{array}{lcl}
Expr & \rightarrow & Expr \ \texttt{+} \ Term \\
 & | & Term \\
Term & \rightarrow & Term \ \star \ Factor \\
 & | & Factor \\
Factor & \rightarrow & \texttt{(} \, Expr \, \texttt{)} \\
 & | & \texttt{a}
\end{array}
$$

  Trace *TDParse* on how to build an AST for input `a + a * a`.
- Running *TDParse* with **G** results an ***infinite loop*** !!!
  - *TDParse* focuses on the ***leftmost*** non-terminal.
  - The grammar **G** contains ***left-recursions***.
- We must first convert left-recursions in **G** to ***right-recursions***.

TDP: Exercise (2)

- Given the following CFG **G**:

$$
\begin{array}{lcl}
Expr & \rightarrow & Term \ \ Expr' \\
Expr' & \rightarrow & \texttt{+} \ Term \ \ Expr' \\
 & | & \epsilon \\
Term & \rightarrow & Factor \ \ Term' \\
Term' & \rightarrow & \star \ Factor \ \ Term' \\
 & | & \epsilon \\
Factor & \rightarrow & \texttt{(} \, Expr \, \texttt{)} \\
 & | & \texttt{a}
\end{array}
$$

  Exercise. Trace *TDParse* on building AST for `a + a * a`.

  Exercise. Trace *TDParse* on building AST for `(a + a) * a`.

  Q: How to handle ϵ-productions (e.g., $Expr \rightarrow \epsilon$)?

  A: Execute *focus* `:=` *trace*`.pop()` to advance to next node.
- Running *TDParse* will ***terminate*** $\because$ **G** is ***right-recursive***.
- We will learn about a systematic approach to converting left-recursions in a given grammar to ***right-recursions***.

Left-Recursions (LR): Direct vs. Indirect

Given CFG $G = (V, \Sigma, R, S)$, $\alpha, \beta, \gamma \in (V \cup \Sigma)^*$, G contains:

- A *cycle* if $\exists A \in V \bullet A \stackrel{*}{\Rightarrow} A$
- A ***direct*** LR if $A \rightarrow A\alpha \in R$ for non-terminal $A \in V$

e.g.,

$$\begin{array}{lcl} Expr & \rightarrow & Expr + Term \\ & | & Term \\ Term & \rightarrow & Term * Factor \\ & | & Factor \\ Factor & \rightarrow & (Expr) \\ & | & \mathtt{a} \end{array}$$

e.g.,

$$\begin{array}{lcl} Expr & \rightarrow & Expr + Term \\ & | & Expr - Term \\ & | & Term \\ Term & \rightarrow & Term * Factor \\ & | & Term \,/\, Factor \\ & | & Factor \end{array}$$

- An ***indirect*** LR if $A \rightarrow B\beta \in R$ for non-terminals $A, B \in V$, $B \stackrel{*}{\Rightarrow} A\gamma$

$$\begin{array}{lcl} A & \rightarrow & Br \\ B & \rightarrow & Cd \\ C & \rightarrow & At \end{array}$$

$A \rightarrow Br, B \stackrel{*}{\Rightarrow} Atd$

$$\begin{array}{lclclcl} A & \rightarrow & Ba & | & b & & \\ B & \rightarrow & Cd & | & e & & \\ C & \rightarrow & Df & | & g & & \\ D & \rightarrow & f & | & Aa & | & Cg \end{array}$$

$A \rightarrow Ba, B \stackrel{*}{\Rightarrow} Aafd$

TDP: (Preventively) Eliminating LRs

```
1   ALGORITHM: RemoveLR
2     INPUT: CFG G = (V, Σ, R, S)
3     ASSUME: G has no ε-productions
4     OUTPUT: G′ s.t. G′ ≡ G, G′ has no
5             indirect & direct left-recursions
6   PROCEDURE:
7     impose an order on V: ⟨⟨A1, A2, ..., An⟩⟩
8     for i: 1 .. n:
9       for j: 1 .. i − 1:
10        if ∃ Ai → Ajγ ∈ R ∧ Aj → δ1 | δ2 | ... | δm ∈ R then
11          replace Ai → Ajγ with Ai → δ1γ | δ2γ | ... | δmγ
12        end
13      for Ai → Aiα | β ∈ R:
14        replace it with: Ai → βAi′, Ai′ → αAi′ | ε
```

- **L9** to **L12**: Remove ***indirect*** left-recursions from A_1 to A_{i-1}.
- **L13** to **L14**: Remove ***direct*** left-recursions from A_1 to A_{i-1}.
- ***Loop Invariant*** **(outer for-loop)**? At the start of i^{th} iteration:
  - No ***direct*** or ***indirect*** left-recursions for $A_1, A_2, \dots, A_{i-1}$.
  - More precisely: $\forall j : j < i \bullet \neg(\exists k \bullet k \le j \wedge A_j \rightarrow A_k \cdots \in R)$

CFG: Eliminating ϵ-Productions (1)

- Motivations:
  - ***TDParse*** handles each ϵ-production as a special case.
  - ***RemoveLR*** produces CFG which may contain ϵ-productions.
- $\epsilon \notin L \Rightarrow \exists$ CFG $G = (V, \Sigma, R, S)$ s.t. G has no ϵ-productions.

  An *ϵ-production* has the form $A \rightarrow \epsilon$.
- A variable A is ***nullable*** if $A \stackrel{*}{\Rightarrow} \epsilon$.
  - Each terminal symbol is ***not nullable***.
  - Variable A is ***nullable*** if either:
    - $A \rightarrow \epsilon \in R$; or
    - $A \rightarrow B_1 B_2 \dots B_k \in R$, where each variable B_i $(1 \le i \le k)$ is a ***nullable***.
- Given a production $B \rightarrow CAD$, if only variable A is ***nullable***, then there are 2 versions of B: $B \rightarrow CAD \mid CD$
- In general, given a production $A \rightarrow X_1 X_2 \dots X_k$ with k symbols, if m of the k symbols are ***nullable***:
  - $m < k$: There are 2^m versions of A.
  - $m = k$: There are $2^m - 1$ versions of A. [excluding $A \rightarrow \epsilon$]

CFG: Eliminating ϵ-Productions (2)

- Eliminate ϵ-productions from the following grammar:

$$\begin{array}{lcl} S & \rightarrow & AB \\ A & \rightarrow & aAA \mid \epsilon \\ B & \rightarrow & bBB \mid \epsilon \end{array}$$

- Which are the ***nullable*** variables? [S, A, B]

$$\begin{array}{lcll} S & \rightarrow & A \mid B \mid AB & \{S \rightarrow \epsilon \texttt{ not included}\} \\ A & \rightarrow & aAA \mid aA \mid a & \{A \rightarrow aA \texttt{ duplicated}\} \\ B & \rightarrow & bBB \mid bB \mid b & \{B \rightarrow bB \texttt{ duplicated}\} \end{array}$$

Backtrack-Free Parsing (1)

- TDParse automates the ***top-down***, ***leftmost*** derivation process by consistently choosing production rules (e.g., in order of their appearance in CFG).
  - This ***inflexibility*** may lead to ***inefficient*** runtime performance due to the need to *backtrack*.
  - e.g., It may take the ***construction of a giant subtree*** to find out a ***mismatch*** with the input tokens, which end up requiring it to *backtrack* all the way back to the ***root*** (start symbol).
- We may avoid backtracking with a modification to the parser:
  - When deciding which production rule to choose, consider:
    (1) the ***current*** input symbol
    (2) the consequential ***first*** symbol if a rule was applied for focus
    [*lookahead* symbol]
  - Using a ***one symbol lookhead***, w.r.t. a ***right-recursive*** CFG, each alternative for the ***leftmost nonterminal*** leads to a ***unique terminal***, allowing the parser to decide on a choice that prevents *backtracking*.
  - Such CFG is ***backtrack free*** with the ***lookhead*** of one symbol.
  - We also call such backtrack-free CFG a ***predictive grammar***.

The FIRST Set: Definition

- Say we write $T \subset \mathbb{P}(\Sigma^*)$ to denote the set of valid tokens recognizable by the scanner.
- FIRST$(\alpha) \triangleq$ set of symbols that can appear as the ***first word*** in some string derived from α.
- More precisely:

$$\text{FIRST}(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in T \\ \{w \mid w \in \Sigma^* \land \alpha \overset{*}{\Rightarrow} w\beta \land \beta \in (V \cup \Sigma)^*\} & \text{if } \alpha \in V \end{cases}$$

The FIRST Set: Examples

- Consider this ***right***-recursive CFG:

0	*Goal*	→	*Expr*		6	*Term′*	→	x *Factor Term′*
1	*Expr*	→	*Term Expr′*		7		|	÷ *Factor Term′*
2	*Expr′*	→	+ *Term Expr′*		8		|	ϵ
3		|	- *Term Expr′*		9	*Factor*	→	(*Expr*)
4		|	ϵ		10		|	num
5	*Term*	→	*Factor Term′*		11		|	name

- Compute FIRST for each terminal (e.g., num, +, ( ):

	num	name	+	-	×	÷	(	)	eof	ϵ
FIRST	num	name	+	-	x	÷	(	)	eof	ϵ

- Compute FIRST for each non-terminal (e.g., *Expr*, *Term′*):

	Expr	*Expr′*	*Term*	*Term′*	*Factor*
FIRST	(, name, num	+, -, ϵ	(, name, num	x, ÷, ϵ	(, name, num

Computing the FIRST Set

$$\text{FIRST}(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha \in T \\ \{w \mid w \in \Sigma^* \land \alpha \overset{*}{\Rightarrow} w\beta \land \beta \in (V \cup \Sigma)^*\} & \text{if } \alpha \in V \end{cases}$$

```
ALGORITHM: GetFirst
  INPUT: CFG G = (V, Σ, R, S)
  T ⊂ Σ* denotes valid terminals
  OUTPUT: FIRST : V ∪ T ∪ {ε, eof} ⟶ ℙ(T ∪ {ε, eof})
PROCEDURE:
  for α ∈ (T ∪ {eof, ε}): FIRST(α) := {α}
  for A ∈ V: FIRST(A) := ∅
  lastFirst := ∅
  while (lastFirst ≠ FIRST):
    lastFirst := FIRST
    for A → β1β2...βk ∈ R s.t. ∀βj : βj ∈ (T ∪ V):
      rhs := FIRST(β1) − {ε}
      for (i := 1; ε ∈ FIRST(βi) ∧ i < k; i++):
        rhs := rhs ∪ (FIRST(βi+1) − {ε})
      if i = k ∧ ε ∈ FIRST(βk) then
        rhs := rhs ∪ {ε}
      end
      FIRST(A) := FIRST(A) ∪ rhs
```

Computing the FIRST Set: Extension

- Recall: **FIRST** takes as input a token or a variable.

$$\boxed{\textbf{FIRST} : V \cup T \cup \{\epsilon, eof\} \longrightarrow \mathbb{P}(T \cup \{\epsilon, eof\})}$$

- The computation of variable ***rhs*** in algoritm `GetFirst` actually suggests an extended, overloaded version:

$$\boxed{\textbf{FIRST} : (V \cup T \cup \{\epsilon, eof\})^* \longrightarrow \mathbb{P}(T \cup \{\epsilon, eof\})}$$

FIRST may also take as input a string $\beta_1\beta_2\dots\beta_n$ (RHS of rules).

- More precisely:

$$\textbf{FIRST}(\beta_1\beta_2\dots\beta_n) = \left\{ \textbf{FIRST}(\beta_1) \cup \textbf{FIRST}(\beta_2) \cup \dots \cup \textbf{FIRST}(\beta_{k-1}) \cup \textbf{FIRST}(\beta_k) \;\middle|\; \begin{array}{l} \forall i : 1 \leq i < k \bullet \epsilon \in \textbf{FIRST}(\beta_i) \\ \wedge \\ \epsilon \notin \textbf{FIRST}(\beta_k) \end{array} \right.$$

Note. β_k is the first symbol whose **FIRST** set does not contain ϵ.

Extended FIRST Set: Examples

Consider this ***right***-recursive CFG:

0	*Goal*	→	*Expr*	6	*Term′*	→	× *Factor Term′*
1	*Expr*	→	*Term Expr′*	7		|	÷ *Factor Term′*
2	*Expr′*	→	+ *Term Expr′*	8		|	ϵ
3		|	- *Term Expr′*	9	*Factor*	→	(*Expr*)
4		|	ϵ	10		|	num
5	*Term*	→	*Factor Term′*	11		|	name

e.g., **FIRST**(*Term Expr′*) = **FIRST**(*Term*) = {(, name, num}

e.g., **FIRST**(+ *Term Expr′*) = **FIRST**(+) = {+}

e.g., **FIRST**(− *Term Expr′*) = **FIRST**(−) = {−}

e.g., **FIRST**(ϵ) = {ϵ}

Is the FIRST Set Sufficient

- Consider the following three productions:

Expr′	→	+	*Term*	*Term′*	(1)
	|	−	*Term*	*Term′*	(2)
	|	ϵ			(3)

In TDP, when the parser attempts to expand an *Expr′* node, it ***looks ahead with one symbol*** to decide on the choice of rule: **FIRST**(+) = {+}, **FIRST**(−) = {−}, and **FIRST**(ϵ) = {ϵ}.

Q. When to choose rule (3) (causing ***focus := trace.pop()***)?

A?. Choose rule (3) when $focus \neq$ **FIRST**(+) $\wedge$ $focus \neq$ **FIRST**(−)?

- ***Correct*** but ***inefficient*** in case of illegal input string: syntax error is only reported after possibly a long series of ***backtrack***.
- Useful if parser knows which words can appear, after an application of the ϵ-production (rule (3)), as leadling symbols.

- **FOLLOW** ($v : V$) $\triangleq$ set of symbols that can appear to the immediate right of a string derived from v.

$$\textbf{FOLLOW}(v) = \{\boldsymbol{w} \mid w, x, y \in \Sigma^* \wedge v \stackrel{*}{\Rightarrow} \boldsymbol{x} \wedge S \stackrel{*}{\Rightarrow} \boldsymbol{xwy}\}$$

The FOLLOW Set: Examples

- Consider this ***right***-recursive CFG:

0	*Goal*	→	*Expr*	6	*Term′*	→	× *Factor Term′*
1	*Expr*	→	*Term Expr′*	7		|	÷ *Factor Term′*
2	*Expr′*	→	+ *Term Expr′*	8		|	ϵ
3		|	- *Term Expr′*	9	*Factor*	→	(*Expr*)
4		|	ϵ	10		|	num
5	*Term*	→	*Factor Term′*	11		|	name

- Compute **FOLLOW** for each non-terminal (e.g., *Expr*, *Term′*):

	Expr	*Expr′*	*Term*	*Term′*	*Factor*
FOLLOW	eof,)	eof,)	eof, +, -,)	eof, +, -,)	eof, +, -, ×, ÷,)

Computing the FOLLOW Set

$\text{FOLLOW}(v) = \{\mathbf{w} \mid w, x, y \in \Sigma^* \land v \overset{*}{\Rightarrow} \mathbf{x} \land S \overset{*}{\Rightarrow} \mathbf{xwy}\}$

```
ALGORITHM: GetFollow
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: FOLLOW : V ⟶ ℙ(T ∪ {eof})
PROCEDURE:
  for A ∈ V: FOLLOW(A) := ∅
  FOLLOW(S) := {eof}
  lastFollow := ∅
  while(lastFollow ≠ FOLLOW):
    lastFollow := FOLLOW
    for A → β1β2...βk ∈ R:
      trailer := FOLLOW(A)
      for i: k .. 1:
        if βi ∈ V then
          FOLLOW(βi) := FOLLOW(βi) ∪ trailer
          if ϵ ∈ FIRST(βi)
            then trailer := trailer ∪ (FIRST(βi) − ϵ)
            else trailer := FIRST(βi)
        else
          trailer := FIRST(βi)
```


Backtrack-Free Grammar

- A ***backtrack-free grammar*** (for a **top-down parser**), when expanding the ***focus internal node***, is always able to choose a unique rule with the ***one-symbol lookahead*** (or report a ***syntax error*** when no rule applies).
- To formulate this, we first define:

$$\text{START}(A \to \beta) = \begin{cases} \text{FIRST}(\beta) & \text{if } \epsilon \notin \text{FIRST}(\beta) \\ \text{FIRST}(\beta) \cup \text{FOLLOW}(A) & \text{otherwise} \end{cases}$$

  $\text{FIRST}(\beta)$ is the extended version where β may be $\beta_1\beta_2\dots\beta_n$
- A ***backtrack-free grammar*** has each of its productions $A \to \gamma_1 \mid \gamma_2 \mid \dots \mid \gamma_n$ satisfying:

$$\forall i, j : 1 \le i, j \le n \land i \ne j \bullet \text{START}(\gamma_i) \cap \text{START}(\gamma_j) = \varnothing$$

TDP: Lookahead with One Symbol

```
ALGORITHM: TDParse
  INPUT: CFG G = (V, Σ, R, S)
  OUTPUT: Root of a Parse Tree or Syntax Error
PROCEDURE:
  root := a new node for the start symbol S
  focus := root
  initialize an empty stack trace
  trace.push(null)
  word := NextWord()
  while (true):
    if focus ∈ V then
      if ∃ unvisited rule focus → β1β2...βn ∈ R ∧ word ∈ START(β) then
        create β1, β2 ... βn as children of focus
        trace.push(βnβn−1 ... β2)
        focus := β1
      else
        if focus = S then report syntax error
        else backtrack
    elseif word matches focus then
      word := NextWord()
      focus := trace.pop()
    elseif word = EOF ∧ focus = null then return root
    else backtrack
```

backtrack ≙ pop *focus*.siblings; *focus* := *focus*.parent; *focus*.resetChildren

Backtrack-Free Grammar: Exercise

Is the following CFG ***backtrack free***?

11	*Factor*	→	`name`
12		|	`name` [*ArgList*]
13		|	`name` (*ArgList*)
15	*ArgList*	→	*Expr MoreArgs*
16	*MoreArgs*	→	, *Expr MoreArgs*
17		|	ϵ

- $\epsilon \notin \text{FIRST}(\textit{Factor}) \Rightarrow \text{START}(\textit{Factor}) = \text{FIRST}(\textit{Factor})$
- $\text{FIRST}(\textit{Factor} \to \texttt{name}) = \{\texttt{name}\}$
- $\text{FIRST}(\textit{Factor} \to \texttt{name}\ [\textit{ArgList}\,]) = \{\texttt{name}\}$
- $\text{FIRST}(\textit{Factor} \to \texttt{name}\ (\textit{ArgList}\,)) = \{\texttt{name}\}$

∴ The above grammar is ***not*** backtrack free.
⇒ To expand an AST node of *Factor*, with a ***lookahead*** of `name`, the parser has no basis to choose among rules 11, 12, and 13.

Backtrack-Free Grammar: Left-Factoring

- A CFG is not backtrack free if there exists a ***common prefix*** (`name`) among the RHS of ***multiple*** production rules.
- To make such a CFG ***backtrack-free***, we may transform it using *left factoring*: a process of extracting and isolating ***common prefixes*** in a set of production rules.
  - Identify a common prefix α:

$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \ldots \mid \alpha\beta_n \mid \gamma_1 \mid \gamma_2 \mid \ldots \mid \gamma_j$$

[each of $\gamma_1, \gamma_2, \ldots, \gamma_j$ does not begin with α]

  - Rewrite that production rule as:

$$\begin{array}{lcl} A & \rightarrow & \alpha B \mid \gamma_1 \mid \gamma_2 \mid \ldots \mid \gamma_j \\ B & \rightarrow & \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n \end{array}$$

  - New rule $B \rightarrow \beta_1 \mid \beta_2 \mid \ldots \mid \beta_n$ may also contain ***common prefixes***.
  - Rewriting continues until no common prefixes are identified.

Left-Factoring: Exercise

- Use *left-factoring* to remove all ***common prefixes*** from the following grammar.

11	*Factor*	$\rightarrow$	`name`
12		|	`name` [*ArgList*]
13		|	`name` (*ArgList*)
15	*ArgList*	$\rightarrow$	*Expr MoreArgs*
16	*MoreArgs*	$\rightarrow$	, *Expr MoreArgs*
17		|	ϵ

- Identify common prefix `name` and rewrite rules 11, 12, and 13:

$$\begin{array}{lcl} \textit{Factor} & \rightarrow & \texttt{name} \ \ \textit{Arguments} \\ \textit{Arguments} & \rightarrow & [\ \ \textit{ArgList} \ \ ] \\ & \mid & (\ \ \textit{ArgList} \ \ ) \\ & \mid & \epsilon \end{array}$$

Any more ***common prefixes***? [No]

TDP: Terminating and Backtrack-Free

- Given an arbitrary CFG as input to a ***top-down parser***:
  - **Q.** How do we avoid a ***non-terminating*** parsing process?
    A. Convert left-recursions to right-recursion.
  - **Q.** How do we minimize the need of ***backtracking***?
    A. left-factoring & one-symbol lookahead using **START**
- ***Not*** every context-free language has a corresponding ***backtrack-free*** context-free grammar.

  Given a CFL l, the following is ***undecidable***:

$$\boxed{\exists cfg \mid L(cfg) = l \wedge isBacktrackFree(cfg)}$$

- Given a CFG $g = (V, \Sigma, R, S)$, whether or not g is ***backtrack-free*** is ***decidable***:

  For each $A \rightarrow \gamma_1 \mid \gamma_2 \mid \ldots \mid \gamma_n \in R$:

$$\boxed{\forall i, j : 1 \leq i, j \leq n \wedge i \neq j \bullet \text{START}(\gamma_i) \cap \text{START}(\gamma_j) = \varnothing}$$

Backtrack-Free Parsing (2.1)

- A *recursive-descent* parser is:
  - A top-down parser
  - Structured as a set of *mutually recursive* procedures

    Each procedure corresponds to a ***non-terminal*** in the grammar.

    See an example.
- Given a ***backtrack-free*** grammar, a tool (a.k.a. *parser generator*) can automatically generate:
  - **FIRST**, **FOLLOW**, and **START** sets
  - An efficient ***recursive-descent*** parser

    This generated parser is called an *LL(1) parser*, which:
    - Processes input from Left to right
    - Constructs a Leftmost derivation
    - Uses a lookahead of 1 symbol
- *LL(1) grammars* are those working in an *LL(1)* scheme.

  LL(1) grammars are ***backtrack-free*** by definition.

Backtrack-Free Parsing (2.2)

- Consider this CFG with **START** sets of the RHSs:

	Production	FIRST^+
2	$Expr' \rightarrow + \; Term \; Expr'$	{ + }
3	$\mid \; - \; Term \; Expr'$	{ - }
4	$\mid \; \epsilon$	{ ϵ, eof,) }

- The corresponding ***recursive-descent*** parser is structured as:

```
ExprPrim()
  if word = + ∨ word = - then /* Rules 2, 3 */
    word := NextWord()
    if(Term())
      then return ExprPrim()
      else return false
  elseif word = ) ∨ word = eof then /* Rule 4 */
    return true
  else
    report a syntax error
    return false
  end

Term()
  ...
```

See: parser generator

LL(1) Parser: Exercise

Consider the following grammar:

$$
\begin{array}{lcl@{\qquad}lcl@{\qquad}lcl}
L & \rightarrow & R \; \texttt{a} & R & \rightarrow & \texttt{aba} & Q & \rightarrow & \texttt{bbc} \\
 & \mid & Q \; \texttt{ba} & & \mid & \texttt{caba} & & \mid & \texttt{bc} \\
 & & & & \mid & R \; \texttt{bc} & & &
\end{array}
$$

Q. Is it suitable for a ***top-down predictive*** parser?

- If so, show that it satisfies the *LL(1)* condition.
- If not, identify the problem(s) and correct it (them). Also show that the revised grammar satisfies the *LL(1)* condition.

BUP: Discovering Rightmost Derivation

- In TDP, we build the start variable as the ***root node***, and then work towards the ***leaves***. [**leftmost** derivation]
- In Bottom-Up Parsing (BUP):
  - Words (terminals) are still returned from **left** to **right** by the scanner.
  - As terminals, or a mix of terminals and variables, are identified as *reducible* to some variable *A* (i.e., matching the RHS of some production rule for *A*), then a layer is added.
  - Eventually:
    - ***accept***:
      The ***start variable*** is reduced and **all** words have been consumed.
    - ***reject***:
      The next word is not `eof`, but no further *reduction* can be identified.

Q. Why can BUP find the ***rightmost*** derivation (RMD), if any?

A. BUP discovers steps in a ***RMD*** in its *reverse* order.

BUP: Discovering Rightmost Derivation (1)

- ***table***-driven *LR(1)* parser: an implementation for BUP, which
  - Processes input from **L**eft to right
  - Constructs a **R**ightmost derivation
  - Uses a lookahead of **1** symbol
- A language has the *LR(1)* property if it:
  - Can be parsed in a single **L**eft to right scan,
  - To build a *reversed* **R**ightmost derivation,
  - Using a lookahead of **1** symbol to determine parsing actions.
- Critical step in a ***bottom-up parser*** is to find the ***next*** *handle*.

BUP: Discovering Rightmost Derivation (2)

```
ALGORITHM: BUParse
  INPUT: CFG G = (V, Σ, R, S), Action & Goto Tables
  OUTPUT: Report Parse Success or Syntax Error
PROCEDURE:
  initialize an empty stack trace
  trace.push(0) /* start state */
  word := NextWord()
  while(true)
    state := trace.top()
    act := Action[state, word]
    if act = ``accept'' then
      succeed()
    elseif act = ``reduce based on A → β'' then
      trace.pop() 2 × |β| times /* word + state */
      state := trace.top()
      trace.push(A)
      next := Goto[state, A]
      trace.push(next)
    elseif act = ``shift to s_i'' then
      trace.push(word)
      trace.push(i)
      word := NextWord()
    else
      fail()
```


BUP: Example Tracing (1)

- Consider the following grammar for parentheses:

1	*Goal*	→	*List*
2	*List*	→	*List Pair*
3		|	*Pair*
4	*Pair*	→	(*Pair*)
5		|	( )

- Assume: tables ***Action*** and ***Goto*** constructed accordingly:

State	Action Table: eof	(	)	Goto Table: *List*	*Pair*
0		s 3		1	2
1	acc	s 3			4
2	r 3	r 3			
3		s 6	s 7		5
4	r 2	r 2			
5			s 8		
6		s 6	s 10		9
7	r 5	r 5			
8	r 4	r 4			
9			s 11		
10			r 5		
11			r 4		

In ***Action*** table:

- s_i: shift to state i
- r_j: reduce to the LHS of production #j

BUP: Example Tracing (2.1)

Consider the steps of performing BUP on input ( ) :

Iteration	State	word	Stack	Handle	Action
initial	—	(	\$ 0	— *none* —	—
1	0	(	\$ 0	— *none* —	*shift 3*
2	3	)	\$ 0 (3	— *none* —	*shift 7*
3	7	eof	\$ 0 (3) 7	( )	*reduce 5*
4	2	eof	\$ 0 *Pair* 2	*Pair*	*reduce 3*
5	1	eof	\$ 0 *List* 1	*List*	*accept*

BUP: Example Tracing (2.2)

Consider the steps of performing BUP on input (( )) ( ) :

Iteration	State	word	Stack	Handle	Action
initial	—	(	\$ 0	— *none* —	—
1	0	(	\$ 0	— *none* —	*shift 3*
2	3	(	\$ 0 (3	— *none* —	*shift 6*
3	6	)	\$ 0 (3 (6	— *none* —	*shift 10*
4	10	)	\$ 0 (3 (6) 10	( )	*reduce 5*
5	5	)	\$ 0 (3 *Pair* 5	— *none* —	*shift 8*
6	8	(	\$ 0 (3 *Pair* 5) 8	(*Pair*)	*reduce 4*
7	2	(	\$ 0 *Pair* 2	*Pair*	*reduce 3*
8	1	(	\$ 0 *List* 1	— *none* —	*shift 3*
9	3	)	\$ 0 *List* 1 (3	— *none* —	*shift 7*
10	7	eof	\$ 0 *List* 1 (3) 7	( )	*reduce 5*
11	4	eof	\$ 0 *List* 1 *Pair* 4	*List Pair*	*reduce 2*
12	1	eof	\$ 0 *List* 1	*List*	*accept*

BUP: Example Tracing (2.3)

Consider the steps of performing BUP on input $\boxed{\texttt{())}}$:

Iteration	State	*word*	Stack	Handle	Action
initial	—	(	$ 0	— *none* —	—
1	0	(	$ 0	— *none* —	*shift 3*
2	3	)	$ 0 (3	— *none* —	*shift 7*
3	7	)	$ 0 (3) 7	— *none* —	*error*

LR(1) Items: Definition

- In **LR(1)** parsing, ***Action*** and ***Goto*** tabeles encode legitimate ways (w.r.t. a CFG) for finding ***handles*** (for ***reductions***).
- In a ***table***-driven **LR(1)** parser, the table-construction algorithm represents each potential ***handle*** (for a ***reduction***) with an **LR(1)** item e.g.,

$$[A \rightarrow \beta \bullet \gamma,\ \mathtt{a}]$$

where:

- A ***production rule*** $\boxed{A \rightarrow \beta\gamma}$ is currently being applied.
- A ***terminal symbol*** $\boxed{\mathtt{a}}$ servers as a ***lookahead symbol***.
- A ***placeholder*** $\boxed{\bullet}$ indicates the parser's ***stack top***.
  - ✓ The parser's ***stack*** contains β ("left context").
  - ✓ γ is yet to be matched.
  - Upon matching $\beta\gamma$, if $\mathtt{a}$ matches the current word, then we "replace" $\beta\gamma$ (and their associated states) with A (and its associated state).

LR(1) Items: Scenarios

An *LR(1) item* can denote:

1. **POSSIBILITY** $[A \rightarrow \bullet\beta\gamma,\ \mathtt{a}]$
   - In the current parsing context, an A would be valid.
   - $\bullet$ represents the position of the parser's ***stack top***
   - Recognizing a β next would be one step towards discovering an A.
2. **PARTIAL COMPLETION** $[A \rightarrow \beta \bullet \gamma,\ \mathtt{a}]$
   - The parser has progressed from $[A \rightarrow \bullet\beta\gamma,\ \mathtt{a}]$ by recognizing β.
   - Recognizing a γ next would be one step towards discovering an A.
3. **COMPLETION** $[A \rightarrow \beta\gamma\bullet,\ \mathtt{a}]$
   - Parser has progressed from $[A \rightarrow \bullet\beta\gamma,\ \mathtt{a}]$ by recognizing $\beta\gamma$.
   - $\beta\gamma$ found in a context where an A followed by $\mathtt{a}$ would be valid.
   - If the current input word matches $\mathtt{a}$, then:
     - Current ***complet item*** is a *handle*.
     - Parser can ***reduce*** $\beta\gamma$ to A
     - Accordingly, in the ***stack***, $\beta\gamma$ (and their associated states) are replaced with A (and its associated state).

LR(1) Items: Example (1.1)

Consider the following grammar for parentheses:

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	(*Pair*)
5		|	( )

Initial State: $[Goal \rightarrow \bullet List,\ \mathtt{eof}]$

Desired Final State: $[Goal \rightarrow List\bullet,\ \mathtt{eof}]$

Intermediate States: Subset Construction

Q. Derive all *LR(1) items* for the above grammar.

- $\text{FOLLOW}(List) = \{\mathtt{eof}, (\}$ $\quad$ $\text{FOLLOW}(Pair) = \{\mathtt{eof}, (,)\}$
- For each production $A \rightarrow \beta$, given $\text{FOLLOW}(A)$, *LR(1) items* are:

$$\{\ [A \rightarrow \bullet\beta\gamma,\ \mathtt{a}] \mid a \in \text{FOLLOW}(A)\ \}$$
$$\cup$$
$$\{\ [A \rightarrow \beta \bullet \gamma,\ \mathtt{a}] \mid a \in \text{FOLLOW}(A)\ \}$$
$$\cup$$
$$\{\ [A \rightarrow \beta\gamma\bullet,\ \mathtt{a}] \mid a \in \text{FOLLOW}(A)\ \}$$

LR(1) Items: Example (1.2)

Q. Given production $A \to \beta$ (e.g., $Pair \to$ `(` $Pair$ `)`), how many *LR(1) items* can be generated?

- The current parsing progress (on matching the RHS) can be:
  1. $\bullet$ `(` $Pair$ `)`
  2. `(` $\bullet Pair$ `)`
  3. `(` $Pair \bullet$ `)`
  4. `(` $Pair$ `)` $\bullet$
- Lookahead symbol following $Pair$? **FOLLOW**$(Pair) = \{$`eof`, `(`, `)`$\}$
- All possible *LR(1) items* related to $Pair \to$ `(` $Pair$ `)`?

✓	[$\bullet$($Pair$), eof]	[$\bullet$($Pair$), (]	[$\bullet$($Pair$),)]
✓	[($\bullet Pair$), eof]	[($\bullet Pair$), (]	[($\bullet Pair$),)]
✓	[($Pair \bullet$), eof]	[($Pair \bullet$), (]	[($Pair \bullet$),)]
✓	[($Pair$)$\bullet$, eof]	[($Pair$)$\bullet$, (]	[($Pair$)$\bullet$,)]

A. How many in general (in terms of A and β)?

$$\underbrace{|\beta| + 1}_{\texttt{possible positions of } \bullet} \quad \times \quad \underbrace{|\textbf{FOLLOW}(A)|}_{\texttt{possible lookahead symbols}}$$

LR(1) Items: Example (1.3)

A. There are 33 *LR(1) items* in the parentheses grammar.

[$Goal \to \bullet\, List$,eof]
[$Goal \to List\, \bullet$,eof]

[$List \to \bullet\, List\ Pair$,eof]	[$List \to \bullet\, List\ Pair$,(]
[$List \to List \bullet Pair$,eof]	[$List \to List \bullet Pair$,(]
[$List \to List\ Pair \bullet$,eof]	[$List \to List\ Pair \bullet$,(]
[$List \to \bullet\, Pair$,eof]	[$List \to \bullet\, Pair$,(]
[$List \to Pair \bullet$,eof]	[$List \to Pair \bullet$,(]

[$Pair \to \bullet$ ($Pair$),eof]	[$Pair \to \bullet$ ($Pair$),)]	[$Pair \to \bullet$ ($Pair$),(]
[$Pair \to$ ($\bullet Pair$),eof]	[$Pair \to$ ($\bullet Pair$),)]	[$Pair \to$ ($\bullet Pair$),(]
[$Pair \to$ ($Pair \bullet$),eof]	[$Pair \to$ ($Pair \bullet$),)]	[$Pair \to$ ($Pair \bullet$),(]
[$Pair \to$ ($Pair$) $\bullet$,eof]	[$Pair \to$ ($Pair$) $\bullet$,)]	[$Pair \to$ ($Pair$) $\bullet$,(]
[$Pair \to \bullet$ ( ),eof]	[$Pair \to \bullet$ ( ),(]	[$Pair \to \bullet$ ( ),)]
[$Pair \to$ ($\bullet$),eof]	[$Pair \to$ ($\bullet$),(]	[$Pair \to$ ($\bullet$),)]
[$Pair \to$ ( ) $\bullet$,eof]	[$Pair \to$ ( ) $\bullet$,(]	[$Pair \to$ ( ) $\bullet$,)]

LR(1) Items: Example (2)

Consider the following grammar for expressions:

0	$Goal$	$\to$	$Expr$	6	$Term'$	$\to$	x $Factor\ Term'$
1	$Expr$	$\to$	$Term\ Expr'$	7		|	÷ $Factor\ Term'$
2	$Expr'$	$\to$	+ $Term\ Expr'$	8		|	ϵ
3		|	- $Term\ Expr'$	9	$Factor$	$\to$	($Expr$)
4		|	ϵ	10		|	num
5	$Term$	$\to$	$Factor\ Term'$	11		|	name

Q. Derive all *LR(1) items* for the above grammar.
Hints. First compute **FOLLOW** for each non-terminal:

	Expr	*Expr'*	*Term*	*Term'*	*Factor*
FOLLOW	eof,)	eof,)	eof, +, -,)	eof, +, -,)	eof, +, -, x, ÷,)

Tips. Ignore ϵ ***production*** such as $Expr' \to \epsilon$ since the **FOLLOW** sets already take them into consideration.

Canonical Collection ($\mathcal{CC}$) vs. LR(1) items

1	$Goal$	$\to$	$List$
2	$List$	$\to$	$List\ Pair$
3		|	$Pair$
4	$Pair$	$\to$	($Pair$)
5		|	( )

Recall:
LR(1) Items: 33 items
Initial State: [$Goal \to \bullet List$, eof]
Desired Final State: [$Goal \to List \bullet$, eof]

- The *canonical collection* [Example of $\mathcal{CC}$]
  $$\mathcal{CC} = \{cc_0, cc_1, cc_2, \dots, cc_n\}$$
  denotes the set of **valid subset states** of a **LR(1) parser**.
  - Each $cc_i \in \mathcal{CC}$ $(0 \le i \le n)$ is a set of ***LR(1) items***.
  - $\mathcal{CC} \subseteq \mathbb{P}($***LR(1) items***$)$ $|\mathcal{CC}|$? [$|\mathcal{CC}| \le 2^{|LR(1)\ items|}$]
- To model a **LR(1) parser**, we use techniques analogous to how an ϵ-NFA is converted into a DFA (subset construction and ϵ-closure).
- **Analogies.**
  ✓ ***LR(1) items*** ≈ states of source *NFA*
  ✓ $\mathcal{CC}$ ≈ subset states of target *DFA*

Constructing $\mathcal{CC}$: The *closure* Procedure (1)

```
1   ALGORITHM: closure
2     INPUT: CFG G = (V, Σ, R, S), a set s of LR(1) items
3     OUTPUT: a set of LR(1) items
4   PROCEDURE:
5     lastS := ∅
6     while (lastS ≠ s):
7       lastS := s
8       for [A → ··· • C δ, a] ∈ s:
9         for C → γ ∈ R:
10          for b ∈ FIRST(δa):
11            s := s ∪ { [C → •γ, b] }
12    return s
```

- **Line 8**: $[A \rightarrow \cdots \bullet C\, \delta,\ a] \in s$ indicates that the parser's next task is to match $C\,\delta$ with a lookahead symbol a.
- **Line 9**: Given: matching γ can reduce to C
- **Line 10**: Given: $b \in \textsc{First}(\delta a)$ is a valid lookahead symbol after reducing γ to C
- **Line 11**: Add a new item $[C \rightarrow \bullet\gamma,\ \mathtt{b}]$ into s.
- **Line 6**: Termination is guaranteed.

  $\because$ Each iteration adds ≥ 1 item to s (otherwise $lastS \neq s$ is *false*).

Constructing $\mathcal{CC}$: The *closure* Procedure (2.1)

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	(*Pair*)
5		|	( )

Initial State: $[Goal \rightarrow \bullet List,\ \mathtt{eof}]$

Calculate cc_0 = ***closure***$(\{\ [Goal \rightarrow \bullet List,\ \mathtt{eof}]\ \})$.

Constructing $\mathcal{CC}$: The *goto* Procedure (1)

```
1   ALGORITHM: goto
2     INPUT: a set s of LR(1) items, a symbol x
3     OUTPUT: a set of LR(1) items
4   PROCEDURE:
5     moved := ∅
6     for item ∈ s:
7       if item = [α → β • xδ, a] then
8         moved := moved ∪ { [α → βx • δ, a] }
9       end
10    return closure(moved)
```

Line 7: Given: item $[\alpha \rightarrow \beta \bullet x\delta,\ \mathtt{a}]$ (where x is the next to match)
Line 8: Add $[\alpha \rightarrow \beta x \bullet \delta,\ \mathtt{a}]$ (indicating $\mathtt{x}$ is matched) to *moved*
Line 10: Calculate and return ***closure***(*moved*) as the "***next subset state***" from s with a "transition" $\mathtt{x}$.

Constructing $\mathcal{CC}$: The *goto* Procedure (2)

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	(*Pair*)
5		|	( )

$$\mathrm{cc}_0 = \left\{ \begin{array}{ccc} [Goal \rightarrow \bullet List,\ \mathtt{eof}] & [List \rightarrow \bullet List\ Pair,\ \mathtt{eof}] & [List \rightarrow \bullet List\ Pair,\ \mathtt{(}] \\ [List \rightarrow \bullet Pair,\ \mathtt{eof}] & [List \rightarrow \bullet Pair,\ \mathtt{(}] & [Pair \rightarrow \bullet\ \mathtt{(}\ Pair\ \mathtt{)},\ \mathtt{eof}] \\ [Pair \rightarrow \bullet\ \mathtt{(}\ Pair\ \mathtt{)},\mathtt{(}] & [Pair \rightarrow \bullet\ \mathtt{(}\ \mathtt{)},\ \mathtt{eof}] & [Pair \rightarrow \bullet\ \mathtt{(}\ \mathtt{)},\mathtt{(}] \end{array} \right\}$$

Calculate $goto(cc_0,\ \mathtt{(})$. ["next state" from cc_0 taking $\mathtt{(}$]

Constructing $\mathcal{CC}$: The Algorithm (1)

```
ALGORITHM: BuildCC
  INPUT: a grammar G = (V, Σ, R, S),  goal production S → S'
  OUTPUT:
    (1) a set CC = {cc_0, cc_1, ..., cc_n} where cc_i ⊆ G's LR(1) items
    (2) a transition function
PROCEDURE:
  cc_0 := closure({[S → •S', eof]})
  CC := {cc_0}
  processed := {cc_0}
  lastCC := ∅
  while(lastCC ≠ CC):
    lastCC := CC
    for cc_i s.t. cc_i ∈ CC ∧ cc_i ∉ processed:
      processed := processed ∪ {cc_i}
      for x s.t. [··· → ··· • x ...] ∈ cc_i
        temp := goto(cc_i, x)
        if temp ∉ CC then
          CC := CC ∪ {temp}
        end
        δ := δ ∪ (cc_i, x, temp)
```


Constructing $\mathcal{CC}$: The Algorithm (2.1)

1	*Goal*	$\rightarrow$	*List*
2	*List*	$\rightarrow$	*List Pair*
3		|	*Pair*
4	*Pair*	$\rightarrow$	<u>(</u> *Pair* <u>)</u>
5		|	<u>(</u> <u>)</u>

- Calculate $\mathcal{CC} = \{cc_0, cc_1, \ldots, cc_{11}\}$
- Calculate the transition function $\delta : \mathcal{CC} \times (\Sigma \cup V) \rightarrow \mathcal{CC}$

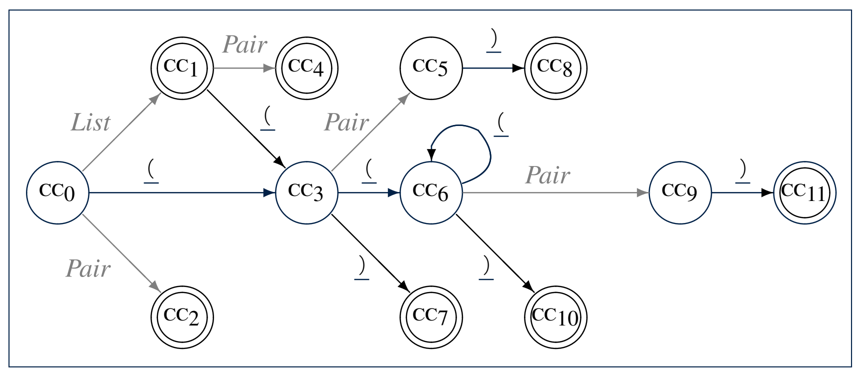
Constructing $\mathcal{CC}$: The Algorithm (2.2)

Resulting transition table:

Iteration	Item	*Goal*	*List*	*Pair*	<u>(</u>	<u>)</u>	eof
0	cc_0	∅	cc_1	cc_2	cc_3	∅	∅
1	cc_1	∅	∅	cc_4	cc_3	∅	∅
	cc_2	∅	∅	∅	∅	∅	∅
	cc_3	∅	∅	cc_5	cc_6	cc_7	∅
2	cc_4	∅	∅	∅	∅	∅	∅
	cc_5	∅	∅	∅	∅	cc_8	∅
	cc_6	∅	∅	cc_9	cc_6	cc_{10}	∅
	cc_7	∅	∅	∅	∅	∅	∅
3	cc_8	∅	∅	∅	∅	∅	∅
	cc_9	∅	∅	∅	∅	cc_{11}	∅
	cc_{10}	∅	∅	∅	∅	∅	∅
4	cc_{11}	∅	∅	∅	∅	∅	∅

Constructing $\mathcal{CC}$: The Algorithm (2.3)

Resulting DFA for the parser:



Constructing $\mathcal{CC}$: The Algorithm (2.4.1)

Resulting canonical collection $\mathcal{CC}$: [Def. of $\mathcal{CC}$]

$$cc_0 = \left\{\begin{matrix} [Goal \rightarrow \bullet\, List,\ \texttt{eof}] & [List \rightarrow \bullet\, List\ Pair,\ \texttt{eof}] & [List \rightarrow \bullet\, List\ Pair,\ \underline{(}] \\ [List \rightarrow \bullet\, Pair,\ \texttt{eof}] & [List \rightarrow \bullet\, Pair,\ \underline{(}] & [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)},\ \texttt{eof}] \\ [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)},\underline{(}] & [Pair \rightarrow \bullet\, \underline{(}\ \underline{)},\ \texttt{eof}] & [Pair \rightarrow \bullet\, \underline{(}\ \underline{)},\underline{(}] \end{matrix}\right\}$$

$$cc_1 = \left\{\begin{matrix} [Goal \rightarrow List\, \bullet,\ \texttt{eof}] & [List \rightarrow List\, \bullet\, Pair,\ \texttt{eof}] & [List \rightarrow List\, \bullet\, Pair,\ \underline{(}] \\ [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)},\ \texttt{eof}] & [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)},\ \underline{(}] & [Pair \rightarrow \bullet\, \underline{(}\ \underline{)},\ \texttt{eof}] \\ & [Pair \rightarrow \bullet\, \underline{(}\ \underline{)},\ \underline{(}] & \end{matrix}\right\}$$

$$cc_2 = \left\{[List \rightarrow Pair\, \bullet,\ \texttt{eof}] \quad [List \rightarrow Pair\, \bullet,\ \underline{(}]\right\}$$

$$cc_3 = \left\{\begin{matrix} [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)},\ \underline{)}] & [Pair \rightarrow \underline{(}\, \bullet\, Pair\ \underline{)},\ \texttt{eof}] & [Pair \rightarrow \underline{(}\, \bullet\, Pair\ \underline{)},\ \underline{(}] \\ [Pair \rightarrow \bullet\, \underline{(}\ \underline{)},\ \underline{)}] & [Pair \rightarrow \underline{(}\, \bullet\, \underline{)},\ \texttt{eof}] & [Pair \rightarrow \underline{(}\, \bullet\, \underline{)},\ \underline{(}] \end{matrix}\right\}$$

$$cc_4 = \left\{[List \rightarrow List\ Pair\, \bullet,\ \texttt{eof}] \quad [List \rightarrow List\ Pair\, \bullet,\ \underline{(}]\right\}$$

$$cc_5 = \left\{[Pair \rightarrow \underline{(}\ Pair\, \bullet\, \underline{)},\ \texttt{eof}] \quad [Pair \rightarrow \underline{(}\ Pair\, \bullet\, \underline{)},\ \underline{(}]\right\}$$

$$cc_6 = \left\{\begin{matrix} [Pair \rightarrow \bullet\, \underline{(}\ Pair\ \underline{)},\ \underline{)}] & [Pair \rightarrow \underline{(}\, \bullet\, Pair\ \underline{)},\ \underline{)}] \\ [Pair \rightarrow \bullet\, \underline{(}\ \underline{)},\ \underline{)}] & [Pair \rightarrow \underline{(}\, \bullet\, \underline{)},\ \underline{)}] \end{matrix}\right\}$$

$$cc_7 = \left\{[Pair \rightarrow \underline{(}\ \underline{)}\, \bullet,\ \texttt{eof}] \quad [Pair \rightarrow \underline{(}\ \underline{)}\, \bullet,\ \underline{(}]\right\}$$

$$cc_8 = \left\{[Pair \rightarrow \underline{(}\ Pair\ \underline{)}\, \bullet,\ \texttt{eof}] \quad [Pair \rightarrow \underline{(}\ Pair\ \underline{)}\, \bullet,\ \underline{(}]\right\}$$

$$cc_9 = \left\{[Pair \rightarrow \underline{(}\ Pair\, \bullet\, \underline{)},\ \underline{)}]\right\}$$

$$cc_{10} = \left\{[Pair \rightarrow \underline{(}\ \underline{)}\, \bullet,\ \underline{)}]\right\}$$

$$cc_{11} = \left\{[Pair \rightarrow \underline{(}\ Pair\ \underline{)}\, \bullet,\ \underline{)}]\right\}$$

Constructing *Action* and *Goto* Tables (1)

```
1  ALGORITHM: BuildActionGotoTables
2    INPUT:
3      (1) a grammar G = (V, Σ, R, S)
4      (2) goal production S → S′
5      (3) a canonical collection CC = {cc_0, cc_1, ..., cc_n}
6      (4) a transition function δ : CC × Σ → CC
7    OUTPUT: Action Table & Goto Table
8  PROCEDURE:
9    for cc_i ∈ CC:
10     for item ∈ cc_i:
11       if item = [A → β • xγ, a] ∧ δ(cc_i, x) = cc_j then
12         Action[i, x] := shift j
13       elseif item = [A → β•, a] then
14         Action[i, a] := reduce A → β
15       elseif item = [S → S′•, eof] then
16         Action[i, eof] := accept
17       end
18     for v ∈ V:
19       if δ(cc_i, v) = cc_j then
20         Goto[i, v] = j
21       end
```

- **L12, 13**: Next valid step in discovering A is to match terminal symbol x.
- **L14, 15**: Having recognized β, if current word matches lookahead a, reduce β to A.
- **L16, 17**: Accept if input exhausted and what's recognized reducible to start var. S.
- **L20, 21**: Record consequence of a reduction to non-terminal v from state i

Constructing *Action* and *Goto* Tables (2)

Resulting **Action** and **Goto** tables:

	Action Table			Goto Table	
State	eof	$\underline{(}$	$\underline{)}$	***List***	***Pair***
0		s 3		1	2
1	acc	s 3			4
2	r 3	r 3			
3		s 6	s 7		5
4	r 2	r 2			
5			s 8		
6		s 6	s 10		9
7	r 5	r 5			
8	r 4	r 4			
9			s 11		
10			r 5		
11			r 4		

BUP: Discovering Ambiguity (1)

1	*Goal*	$\rightarrow$	*Stmt*
2	*Stmt*	$\rightarrow$	`if expr then` *Stmt*
3		|	`if expr then` *Stmt* `else` *Stmt*
4		|	`assign`

- Calculate $\mathcal{CC} = \{cc_0, cc_1, \ldots, \}$
- Calculate the transition function $\delta : \mathcal{CC} \times \Sigma \rightarrow \mathcal{CC}$

BUP: Discovering Ambiguity (2.1)

Resulting transition table:

	Item	*Goal*	*Stmt*	if	expr	then	else	assign	eof
0	cc_0	$\emptyset$	cc_1	cc_2	$\emptyset$	$\emptyset$	$\emptyset$	cc_3	$\emptyset$
1	cc_1	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	cc_2	$\emptyset$	$\emptyset$	$\emptyset$	cc_4	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	cc_3	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
2	cc_4	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	cc_5	$\emptyset$	$\emptyset$	$\emptyset$
3	cc_5	$\emptyset$	cc_6	cc_7	$\emptyset$	$\emptyset$	$\emptyset$	cc_8	$\emptyset$
4	cc_6	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	cc_9	$\emptyset$	$\emptyset$
	cc_7	$\emptyset$	$\emptyset$	$\emptyset$	cc_{10}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	cc_8	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
5	cc_9	$\emptyset$	cc_{11}	cc_2	$\emptyset$	$\emptyset$	$\emptyset$	cc_3	$\emptyset$
	cc_{10}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	cc_{12}	$\emptyset$	$\emptyset$	$\emptyset$
6	cc_{11}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	cc_{12}	$\emptyset$	cc_{13}	cc_7	$\emptyset$	$\emptyset$	$\emptyset$	cc_8	$\emptyset$
7	cc_{13}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	cc_{14}	$\emptyset$	$\emptyset$
8	cc_{14}	$\emptyset$	cc_{15}	cc_7	$\emptyset$	$\emptyset$	$\emptyset$	cc_8	$\emptyset$
9	cc_{15}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

BUP: Discovering Ambiguity (2.2.1)

Resulting canonical collection $\mathcal{CC}$:

$$cc_0 = \left\{ \begin{array}{ll} [Goal \rightarrow \bullet\, Stmt, \texttt{eof}] & [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt, \texttt{eof}] \\ [Stmt \rightarrow \bullet\, \texttt{assign}, \texttt{eof}] & [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt \texttt{ else } Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_1 = \left\{ [Goal \rightarrow Stmt\, \bullet, \texttt{eof}] \right\}$$

$$cc_2 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if} \bullet \texttt{expr then } Stmt, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if} \bullet \texttt{expr then } Stmt \texttt{ else } Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_3 = \left\{ [Stmt \rightarrow \texttt{assign}\, \bullet, \texttt{eof}] \right\}$$

$$cc_4 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr} \bullet \texttt{then } Stmt, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if expr} \bullet \texttt{then } Stmt \texttt{ else } Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_5 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then} \bullet Stmt, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if expr then} \bullet Stmt \texttt{ else } Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{assign}, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_6 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt\, \bullet, \texttt{eof}], \\ [Stmt \rightarrow \texttt{if expr then } Stmt \bullet \texttt{else } Stmt, \texttt{eof}] \end{array} \right\}$$

$$cc_7 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if} \bullet \texttt{expr then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if} \bullet \texttt{expr then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

BUP: Discovering Ambiguity (2.2.2)

Resulting canonical collection $\mathcal{CC}$:

$$cc_8 = \{[Stmt \rightarrow \texttt{assign}\, \bullet, \{\texttt{eof}, \texttt{else}\}]\}$$

$$cc_9 = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt \texttt{ else} \bullet Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt \texttt{ else } Stmt, \texttt{eof}], \\ [Stmt \rightarrow \bullet\, \texttt{assign}, \texttt{eof}] \end{array} \right\}$$

$$cc_{10} = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr} \bullet \texttt{then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if expr} \bullet \texttt{then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_{11} = \{[Stmt \rightarrow \texttt{if expr then } Stmt \texttt{ else } Stmt\, \bullet, \texttt{eof}]\}$$

$$cc_{12} = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then} \bullet Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if expr then} \bullet Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{assign}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_{13} = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt\, \bullet\, , \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if expr then } Stmt \bullet \texttt{else } Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

$$cc_{14} = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt \texttt{ else} \bullet Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{if expr then } Stmt \texttt{ else } Stmt, \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \bullet\, \texttt{assign}, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

BUP: Discovering Ambiguity (3)

- Consider cc_{13}

$$cc_{13} = \left\{ \begin{array}{l} [Stmt \rightarrow \texttt{if expr then } Stmt\, \bullet\, , \{\texttt{eof}, \texttt{else}\}], \\ [Stmt \rightarrow \texttt{if expr then } Stmt \bullet \texttt{else } Stmt, \{\texttt{eof}, \texttt{else}\}] \end{array} \right\}$$

  Q. What does it mean if the current word to consume is `else`?
  A. We can either ***shift*** (then expecting to match another *Stmt*) or ***reduce*** to a *Stmt*.
  Action[13, `else`] cannot hold ***shift*** and ***reduce*** simultaneously.
  $\Rightarrow$ This is known as the *shift-reduce conflict*.

- Consider another scenario:

$$cc_i = \left\{ \begin{array}{l} [A \rightarrow \gamma\delta\bullet,\ \texttt{a}], \\ [B \rightarrow \gamma\delta\bullet,\ \texttt{a}] \end{array} \right\}$$

  Q. What does it mean if the current word to consume is `a`?
  A. We can either ***reduce*** to *A* or ***reduce*** to *B*.
  Action[i, `a`] cannot hold *A* and *B* simultaneously.
  $\Rightarrow$ This is known as the *reduce-reduce conflict*.

Index (1)

Index (2)

Index (3)

Index (4)

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Index (8)