Examples of Arithmetic in Ten's Complement

Add:	−35	plus	+25	equals	−10		
	65	plus	25	equals	90		
Add:	+17	plus	+25	equals	+42		
	17	plus	25	equals	42		
Add:	+20	plus	−30	equals	−10		
	20	plus	70	equals	90		
Add:	−20	plus	+30	equals	+10		
	80	plus	30	equals	10	(110)	

More Examples of Arithmetic in Ten's Complement

```
   +5      05          -4      96
+  -6   +  94       +  +6   +  06
   --      --          --      --
   -1      99          +2      02

   -2      98          -5      95       95
+  -4   +  96       -  +3   -  03    +  97
   --      --          --      --       --
   -6      94          -8               92
```

- subtraction reduces to addition because A – B = A + (–B)
- easy to convert a positive number to its negative counterpart, and a negative number to its positive counterpart.

Two's Complement Representation

- negative numbers: $-x \equiv 2^n - x$
- used in computers because subtraction reduces to addition
  $\Rightarrow$ simpler circuits

To form a negative number:

- start with the positive version of the number
- flip the bits: `0` $\rightarrow$ `1` and `1` $\rightarrow$ `0`
- add `1` to the number produced in the previous step

The above steps are an indirect way of carrying out the evaluation of $2^n - x$, namely $-x \equiv [(2^n - 1) - x] + 1$

Note: same process will convert between positive and negative representations, in both directions.

+5 (`0101`) to −5 (`1011`)

−5 (`1011`) to −(−5) (`0101`)

Example:

$2^4 - x = 2^4 - 5 = 16 - 5 = 11$ in binary this is

$$\begin{array}{r} 10000 \\ -\underline{\quad 101} \\ 1011 \end{array}$$

$(2^4 - 1) - x = (2^4 - 1) - 5 = 10$ in binary this is

$$\begin{array}{r} 1111 \\ -\underline{\quad 101} \\ 1010 \end{array}$$

Now add one to this result.

Example (4-bits, i.e. 4 digit binary)

Side-by-side Comparision: Sign-Magnitude vs. Two's Complement

Natural	Representation	Sign-Magnitude	Two's Complement
0	0000	+0	+0
1	0001	+1	+1
2	0010	+2	+2
3	0011	+3	+3
4	0100	+4	+4
5	0101	+5	+5
6	0110	+6	+6
7	0111	+7	+7
8	1000	not used	-8
9	1001	-1	-7
10	1010	-2	-6
11	1011	-3	-5
12	1100	-4	-4
13	1101	-5	-3
14	1110	-6	-2
15	1111	-7	-1

Big payoff: subtraction reduces to addition

```
     +2 =  0010             +2 =  0010   =  0010
  +  +3 = +0011          -  +3 = -0011   = +1101
     +5 =  0101             -1 =         =  1111

     -2 =  1110             -2 =  1110   =  1110
  +  +3 = +0011          -  +3 = -0011   = +1101
     +1 = 10001             -5 =         = 11011
          |                                |
          throw digit away                 throw digit away
```

Sample Test/Exam questions:

Question:
Convert `-173` to 12-bit two's complement representation. Show all your work.

Answer:

Step 1: convert 173 to binary by repeated division by 2.

```
173 ÷ 2     86   1          1
 86 ÷ 2     43   0         01
 43 ÷ 2     21   1        101
 21 ÷ 2     10   1       1101
 10 ÷ 2      5   0      01101
  5 ÷ 2      2   1     101101
  2 ÷ 2      1   0    0101101
  1 ÷ 2      0   1   10101101
```

Step 2: expand answer in Step 1 to 12-bits

```
000010101101
```

Step 3: flip-the-bits and add one

```
 000010101101

 111101010010
+           1
 ____________
 111101010011
```

Final answer: `-173 = 111101010011`

Question:
Convert `+173` to 12-bit two's complement representation. Show all your work.

Question:
Convert the 12-bit two's complement number `111101010011` to decimal.