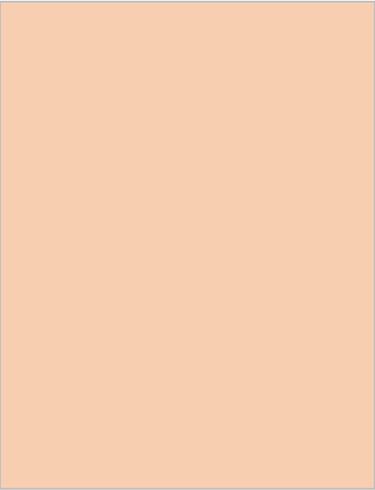


Gene Cheung

Associate Professor, York University

17th June, 2019

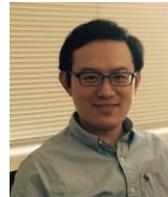


Graph Signal Analysis: Imaging, Learning, Sampling

Acknowledgement

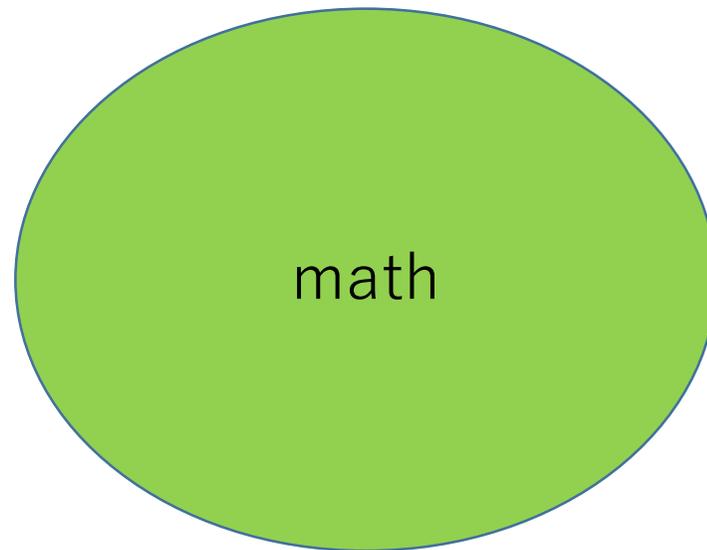
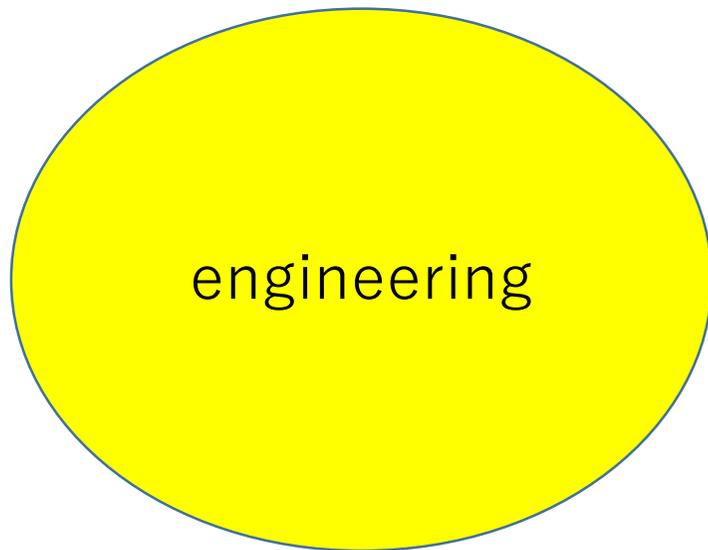
Collaborators:

- **X. Liu** (HIT, China)
- W. Hu, W. Gao (Peking U., China)
- L. Fang (Tsinghua, China)
- Y. Zhao (BJTU, China)
- **C.-W. Lin** (National Tsing Hua University, Taiwan)
- **A. Ortega** (USC, USA)
- D. Florencio (MSR, USA)
- J. Liang, **I. Bajic** (SFU, Canada)
- X. Wu (McMaster U, Canada)
- **P. Frossard** (EPFL, Switzerland)
- V. Stankovic (U of Strathclyde, UK)
- Y. Nakatsukasa (Oxford, UK)
- **P. Le Callet** (U of Nantes, France)



Introducing math tools

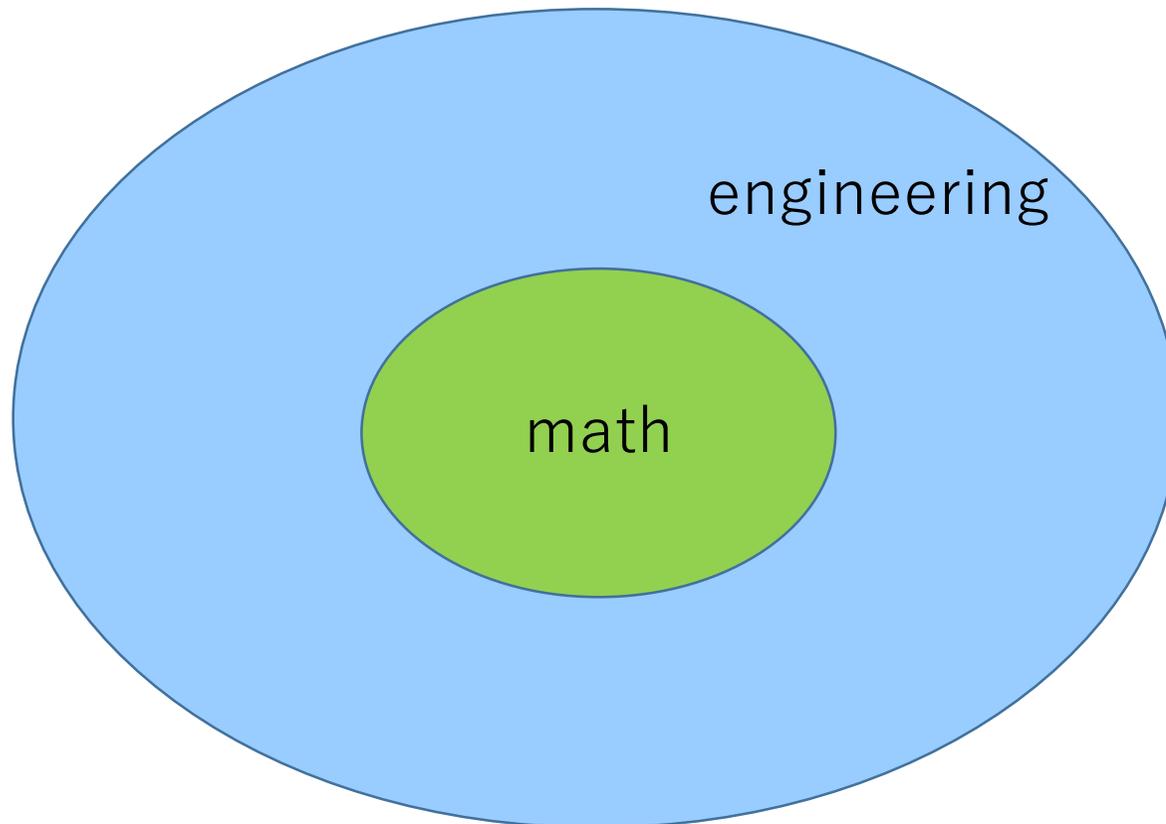
Students in EECS4452: “This is math, not engineering!”



Introducing math tools

Students in EECS4452: “This is math, not engineering!”

Me: “Math is the heart of engineering!”



Outline

- Defining Graph frequencies
- Inverse Imaging
 - Image denoising
 - Image contrast enhancement
 - 3D point cloud denoising / super-resolution
- Deep GLR
- Semi-Supervised Learning
- Graph Sampling
 - Matrix completion

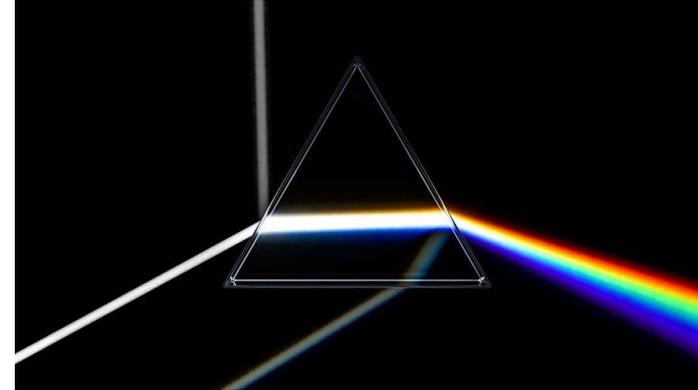
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Signal Decomposition

- Decompose signal into basic components:

$$\mathbf{x} = \sum_{k \in \mathcal{Z}} X_k \varphi_k$$

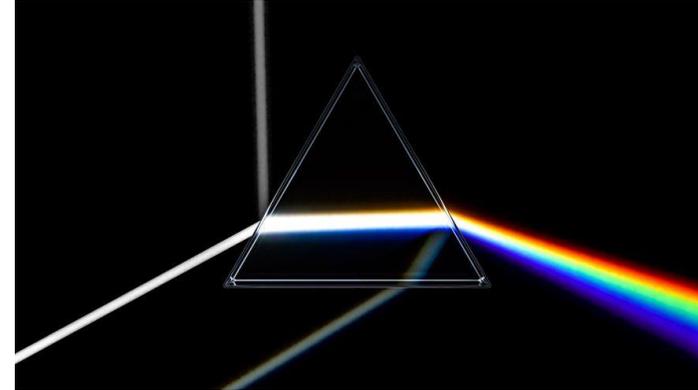


- Newton decomposed white light into color components (1730).

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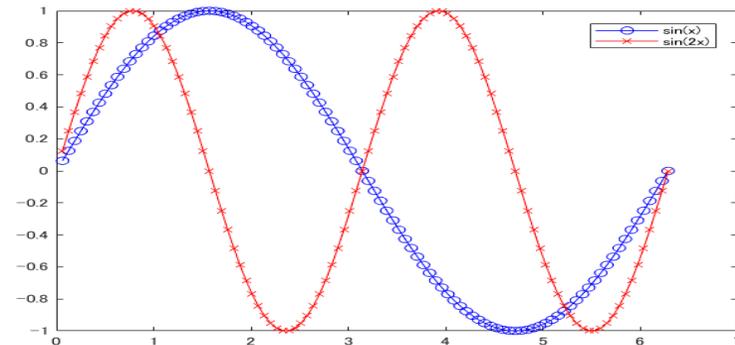


- Newton decomposed white light into color components (1730).

- “Basic” components can be complex exponentials:

$$x = \sum_{k \in \mathbb{Z}} X_k e^{j2\pi kt}$$

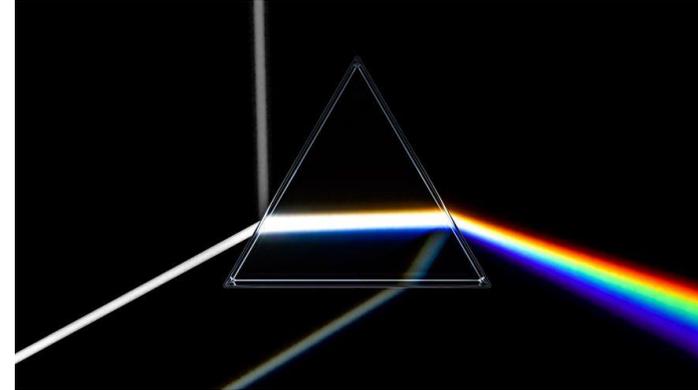
$$X_k = \int x(t) e^{-j2\pi kt} dt$$



Signal Decomposition

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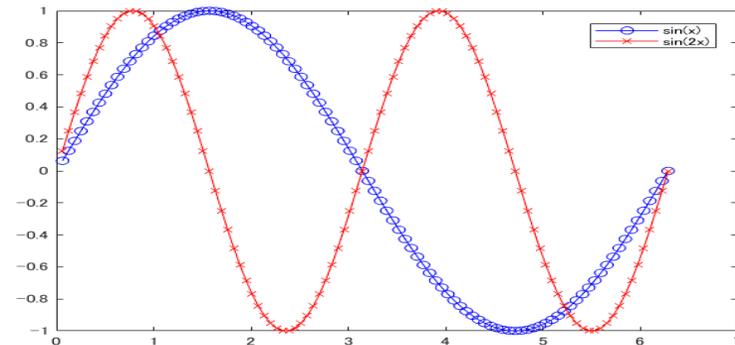


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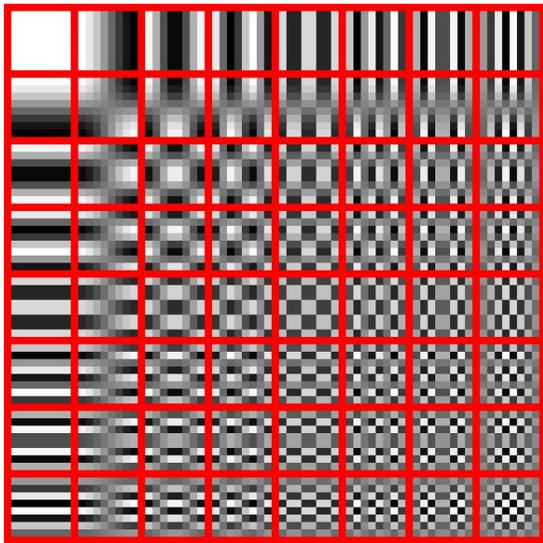
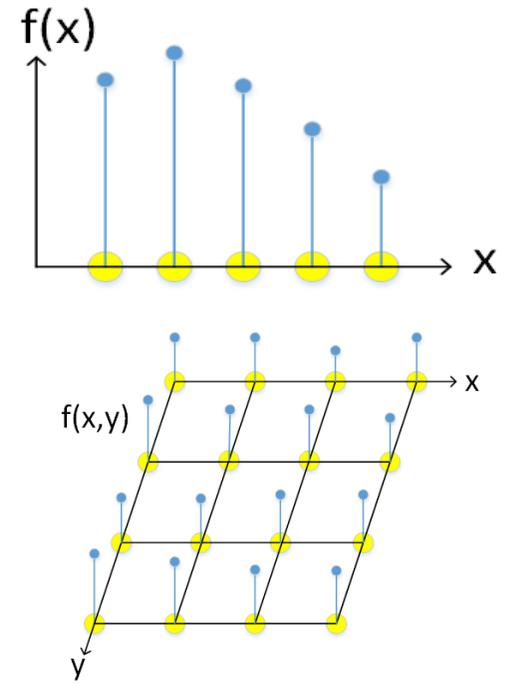
$$X_k = \int x(t) e^{-j2\pi kt} dt$$



- Complex exponentials are eigenfunctions of 2nd derivative operator.

Digital Signal Processing

- Discrete signals on *regular* data kernels.
 - Ex.1: audio on regularly sampled timeline.
 - Ex.2: image on 2D grid.
- **Harmonic analysis** tools (transforms, wavelets):
 - Compression, restoration, segmentation, etc.



2D DCT basis

$$\mathbf{a} = \Phi \mathbf{x}$$

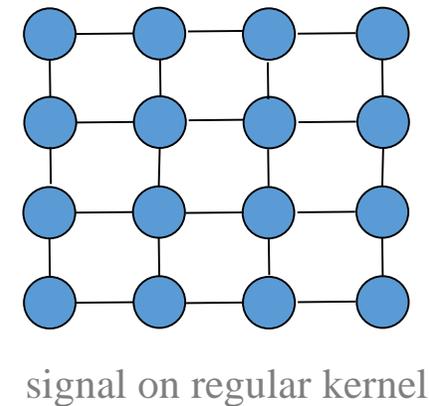
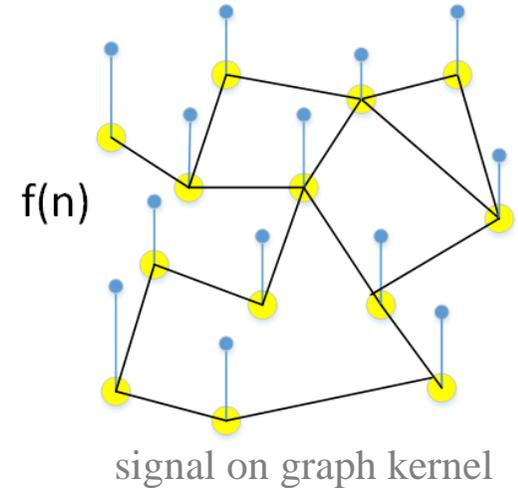
sparse transform coeff. ← desired signal ← transform



Graph Signal Processing

- Signals on *irregular* data kernels described by graphs.
 - Graph: nodes and edges.
 - Edges reveals *node-to-node relationships*.
1. Data domain is naturally a graph.
 2. Embed similarity structure into graph.
 - **Eigenvectors have global info aggregated from local info.**

Graph Signal Processing (GSP) provides spectral analysis tools for signals residing on graphs.



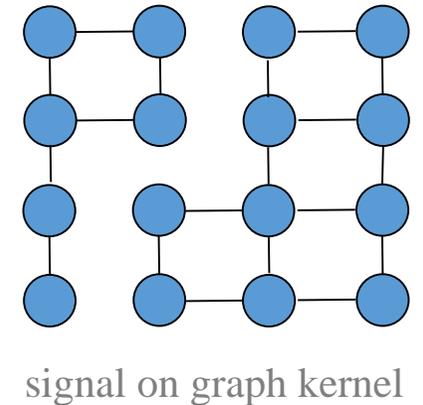
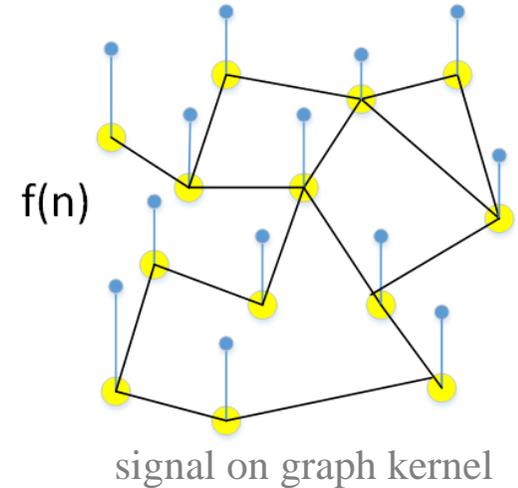
[1] A. Ortega, P. Frossard, J. Kovacevic, J. M. F. Moura, and P. Vandergheynst, "Graph signal processing: Overview, challenges, and applications," *Proceedings of the IEEE*, vol. 106, no. 5, pp. 808–828, 2018.

[2] G. Cheung, E. Magli, Y. Tanaka, and M. K. Ng, "Graph spectral image processing," *Proceedings of the IEEE*, vol. 106, no. 5, pp. 907–930, 2018.

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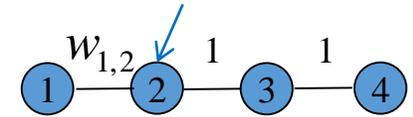


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Graph Fourier Transform (GFT)

undirected graph



$$\mathbf{A} = \begin{bmatrix} 0 & w_{1,2} & 0 & 0 \\ w_{1,2} & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} w_{1,2} & 0 & 0 & 0 \\ 0 & w_{1,2} + 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} w_{1,2} & -w_{1,2} & 0 & 0 \\ -w_{1,2} & w_{1,2} + 1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Graph Laplacian:

- **Adjacency Matrix \mathbf{A}** : entry $A_{i,j}$ has *non-negative* edge weight $w_{i,j}$ connecting nodes i and j .

- **Degree Matrix \mathbf{D}** : diagonal matrix w/ entry $D_{i,i}$ being sum of column entries in row i of \mathbf{A} .

$$D_{i,i} = \sum_j A_{i,j}$$

- **Combinatorial Graph Laplacian \mathbf{L}** : $\mathbf{L} = \mathbf{D} - \mathbf{A}$

- \mathbf{L} is related to *2nd derivative*.

$$\mathbf{L}_{3,:} \mathbf{x} = -x_2 + 2x_3 - x_4$$

$$f''(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

- \mathbf{L} is a differential operator on graph.

Graph Spectrum from GFT

- **Graph Fourier Transform** (GFT) is eigen-matrix of graph Laplacian \mathbf{L} .

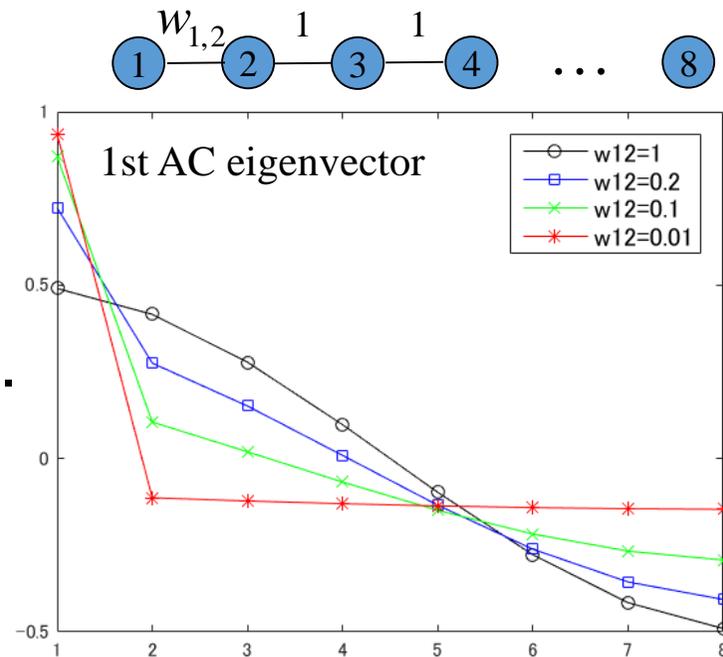
$$\mathbf{L} = \mathbf{V} \mathbf{\Sigma} \mathbf{V}^T$$

eigenvalues along diagonal (pointing to $\mathbf{\Sigma}$)
 eigenvectors in columns (pointing to \mathbf{V})
 GFT (pointing to \mathbf{V}^T)

$$\tilde{\mathbf{X}} = \mathbf{V}^T \mathbf{X}$$

GFT coefficients (pointing to $\tilde{\mathbf{X}}$)

1. Eigenvectors aggregates info from weights.
 - Constant eigenvector is DC.
 - # *zero-crossings* increases as λ increases.
2. Eigenvalues (≥ 0) as *graph frequencies*.



- GFT defaults to *DCT* for un-weighted connected line.
- GFT defaults to *DFT* for un-weighted connected circle.

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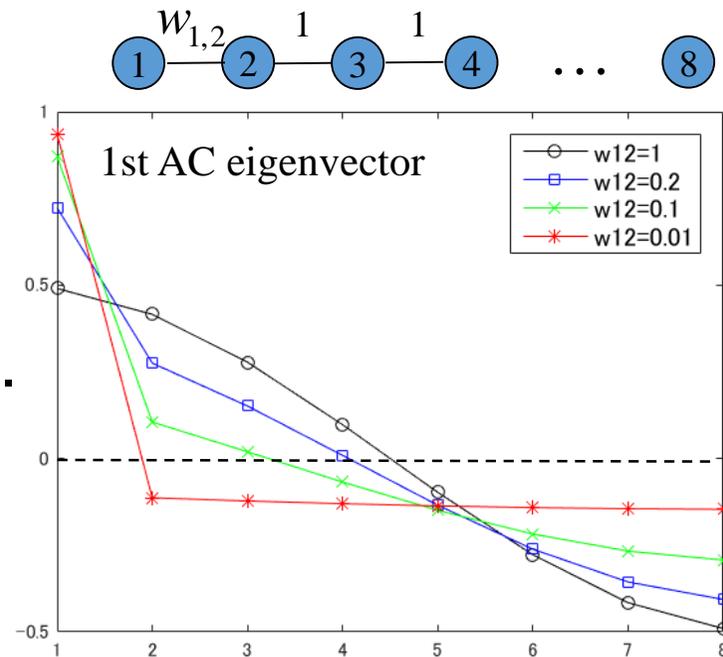
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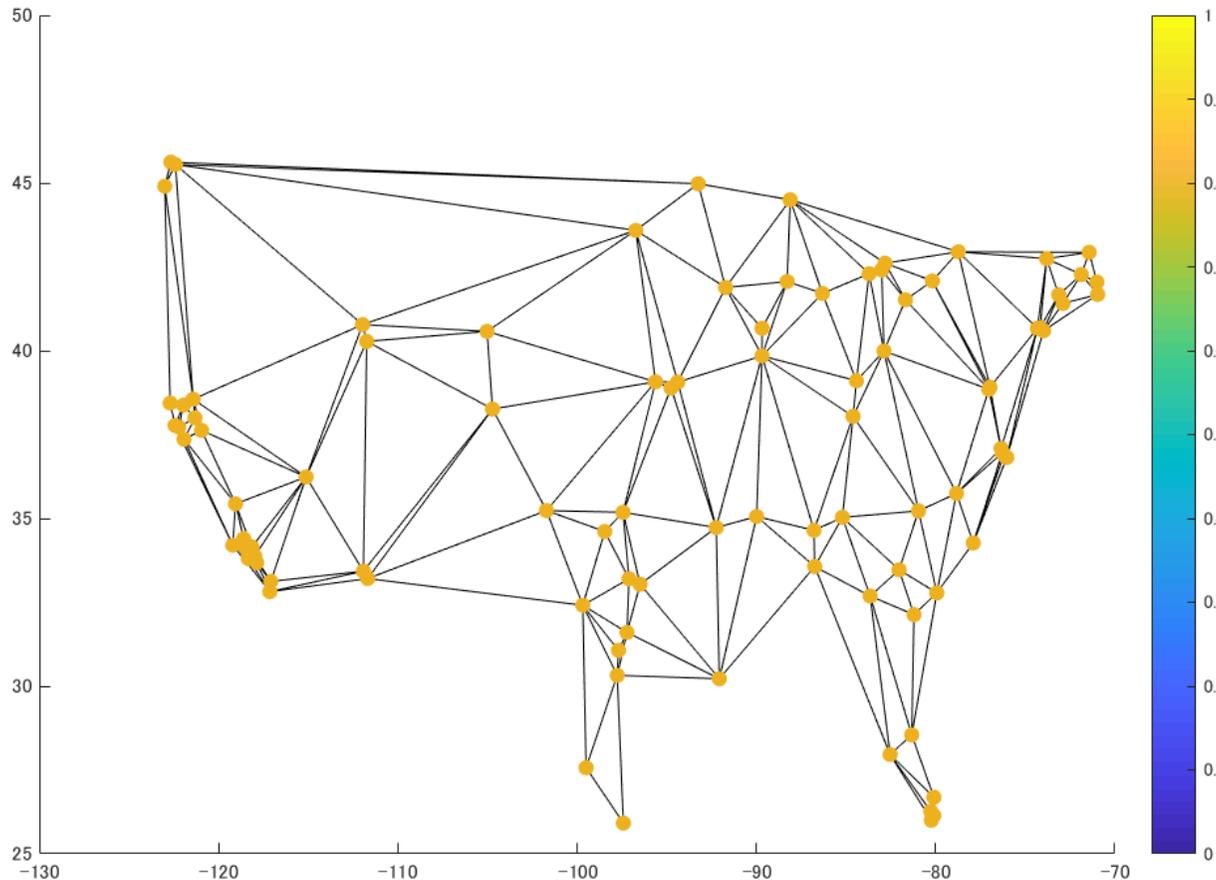
Graph Frequency Examples (US Temperature)

- Weather stations from 100 most populated cities.
- Graph connections from Delaunay Triangulation*.
- Edge weights inverse proportion to distance.

$$w_{i,j} = \exp\left(\frac{-\|l_i - l_j\|_2^2}{\sigma^2}\right)$$

location diff. \swarrow

Edge weights



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V1: DC component

*https://en.wikipedia.org/wiki/Delaunay_triangulation

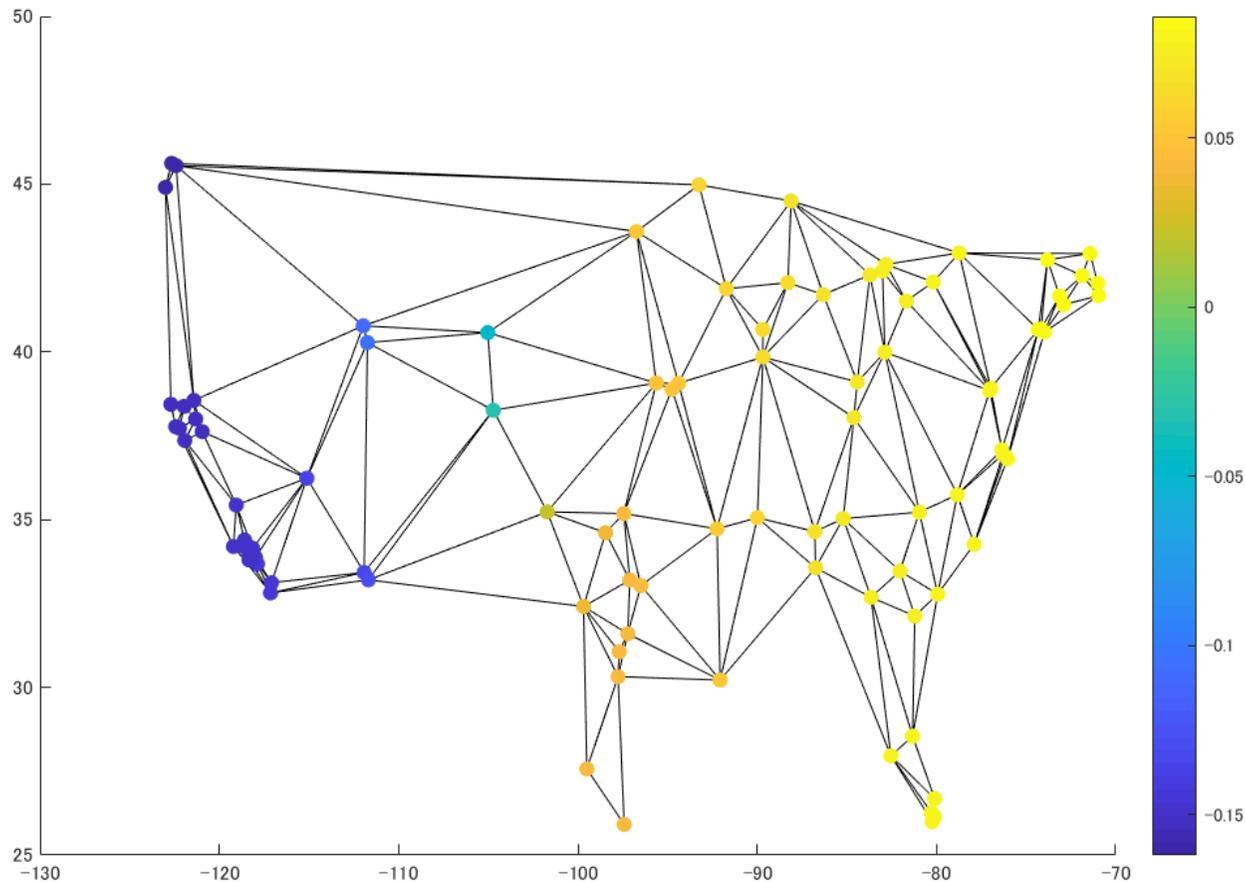
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Edge weights



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V2: 1st AC component

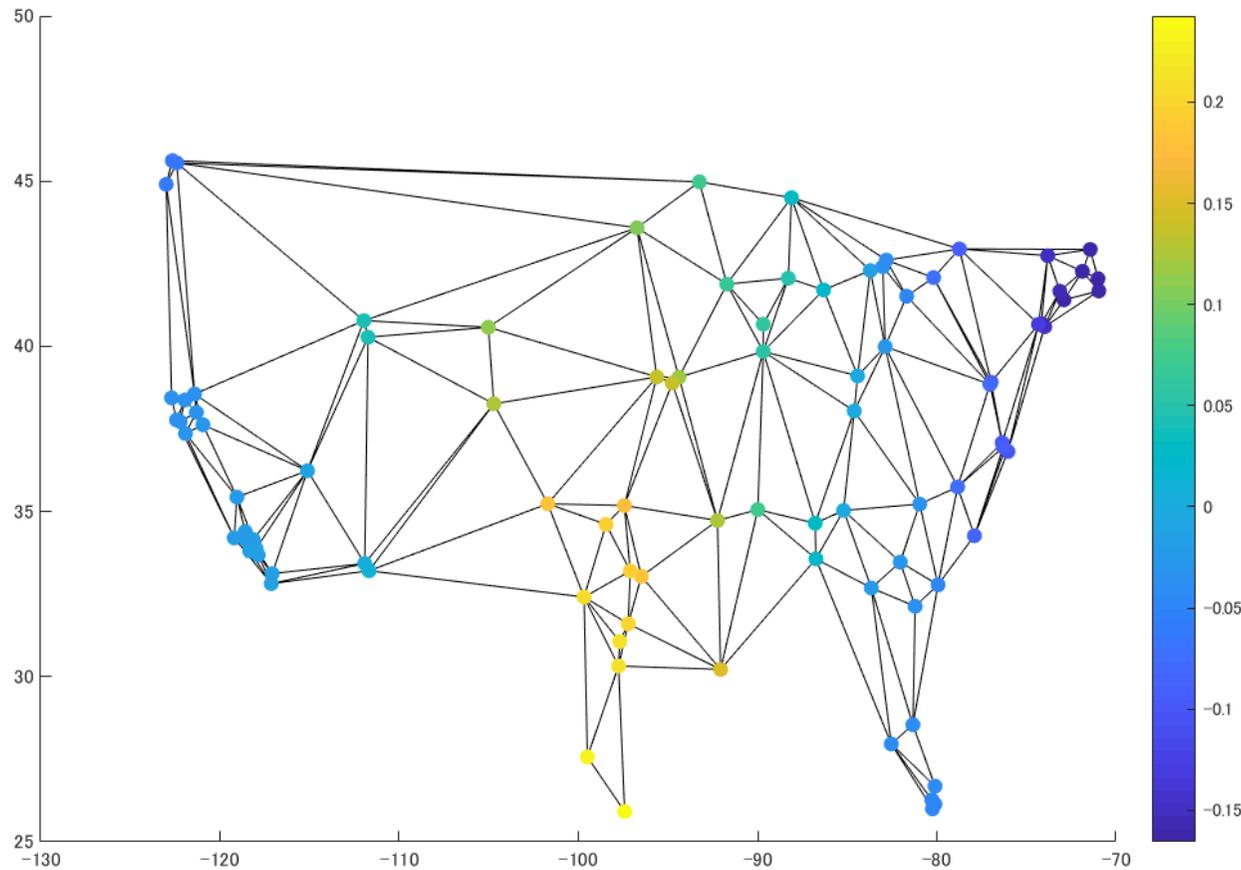
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location diff. \swarrow

Edge weights



V3: 2nd AC component

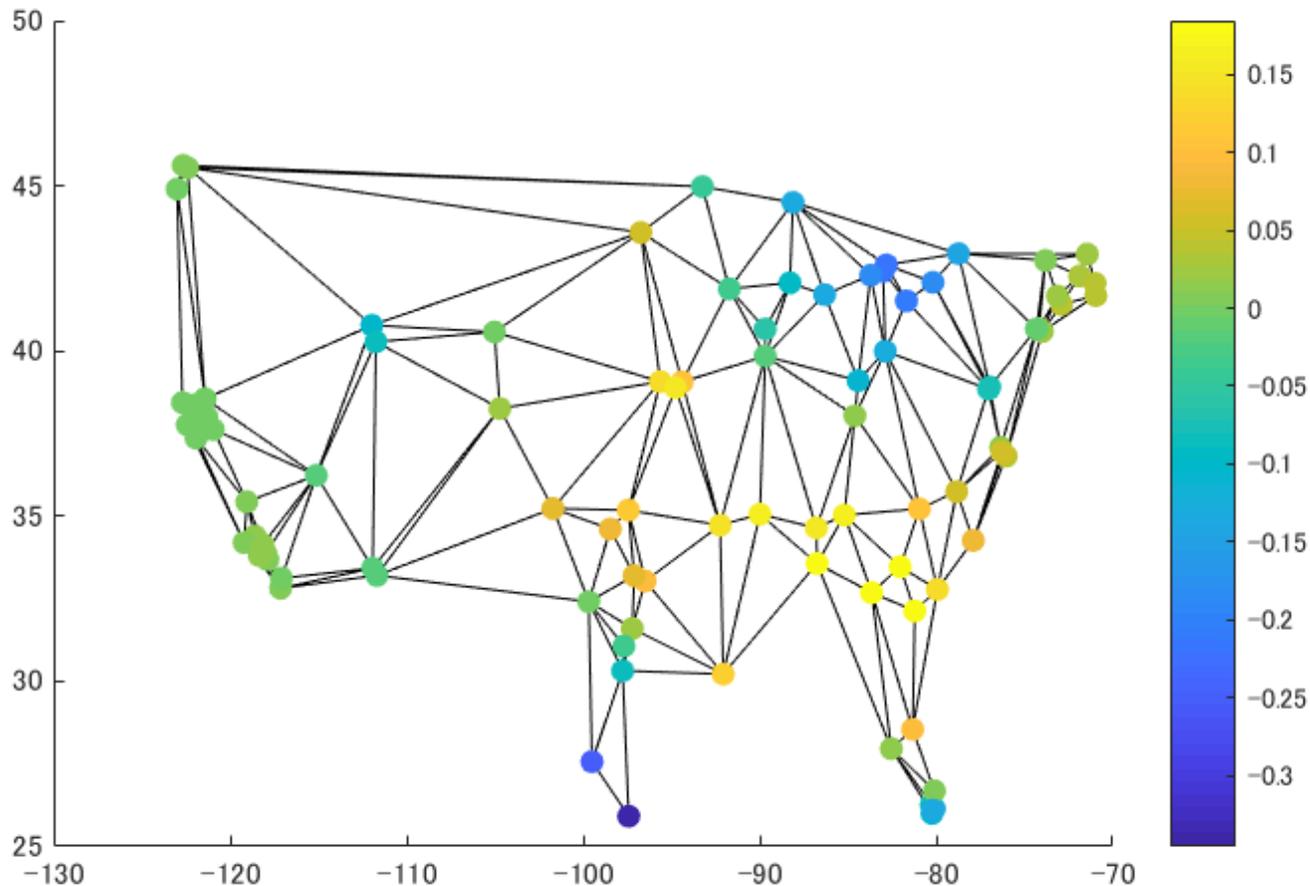
Graph Frequency Examples (US Temperature)

- Weather stations from 100 most populated cities.
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location diff. \swarrow

Edge weights



V4: 9th AC component

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Graph Laplacian Regularizer

- $\mathbf{x}^T \mathbf{L} \mathbf{x}$ (graph Laplacian regularizer) [1]) is one smoothness measure.

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = \frac{1}{2} \sum_{i,j} w_{i,j} (x_i - x_j)^2 = \sum_k \lambda_k \tilde{x}_k^2$$

signal smooth in nodal domain (points to $w_{i,j}$)
 signal contains mostly low graph freq. (points to \tilde{x}_k^2)

- **Signal Denoising:**

observation (points to \mathbf{y})

$$\mathbf{y} = \mathbf{x} + \mathbf{v}$$

desired signal (points to \mathbf{x})
 noise (points to \mathbf{v})

- **MAP Formulation:**

fidelity term (points to $\|y - x\|_2^2$)

$$\min_x \|y - x\|_2^2 + \mu \mathbf{x}^T \mathbf{L} \mathbf{x}$$

smoothness prior (points to $\mu \mathbf{x}^T \mathbf{L} \mathbf{x}$)

$$(\mathbf{I} + \mu \mathbf{L}) \mathbf{x}^* = \mathbf{y}$$

linear system of eqn's w/ sparse, symmetric PD matrix

update edge weights (points to the graph Laplacian context)

Graph Laplacian Regularizer

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signal contains mostly low graph freq.

- **Signal Denoising:** signal smooth in nodal domain

- **MAP Formulation:** $\mathbf{y} = \mathbf{x} + \mathbf{v}$

pixel intensity diff. pixel location diff.

$$w_{i,j} = \exp\left(\frac{-\|x_i - x_j\|_2^2}{\sigma_1^2}\right) \exp\left(\frac{-\|l_i - l_j\|_2^2}{\sigma_2^2}\right)$$

Bilateral filter weights

$$\min_x \|\mathbf{y} - \mathbf{x}\|_2^2 + \mu \mathbf{x}^T \mathbf{L} \mathbf{x}$$

$$(\mathbf{I} + \mu \mathbf{L}) \mathbf{x}^* = \mathbf{y}$$

linear system of eqn's w/ sparse, symmetric PD matrix

update edge weights

Results: natural image denoising

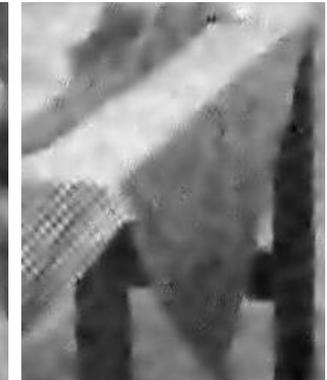
- Subjective comparisons ($\sigma_1 = 40$)



Original



Noisy, 16.48 dB



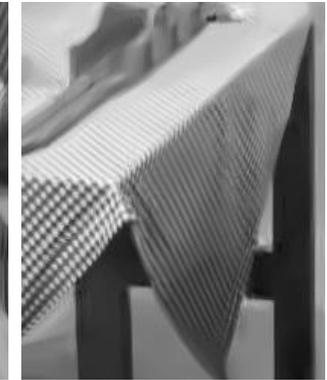
K-SVD, 26.84 dB



BM3D, 27.99 dB



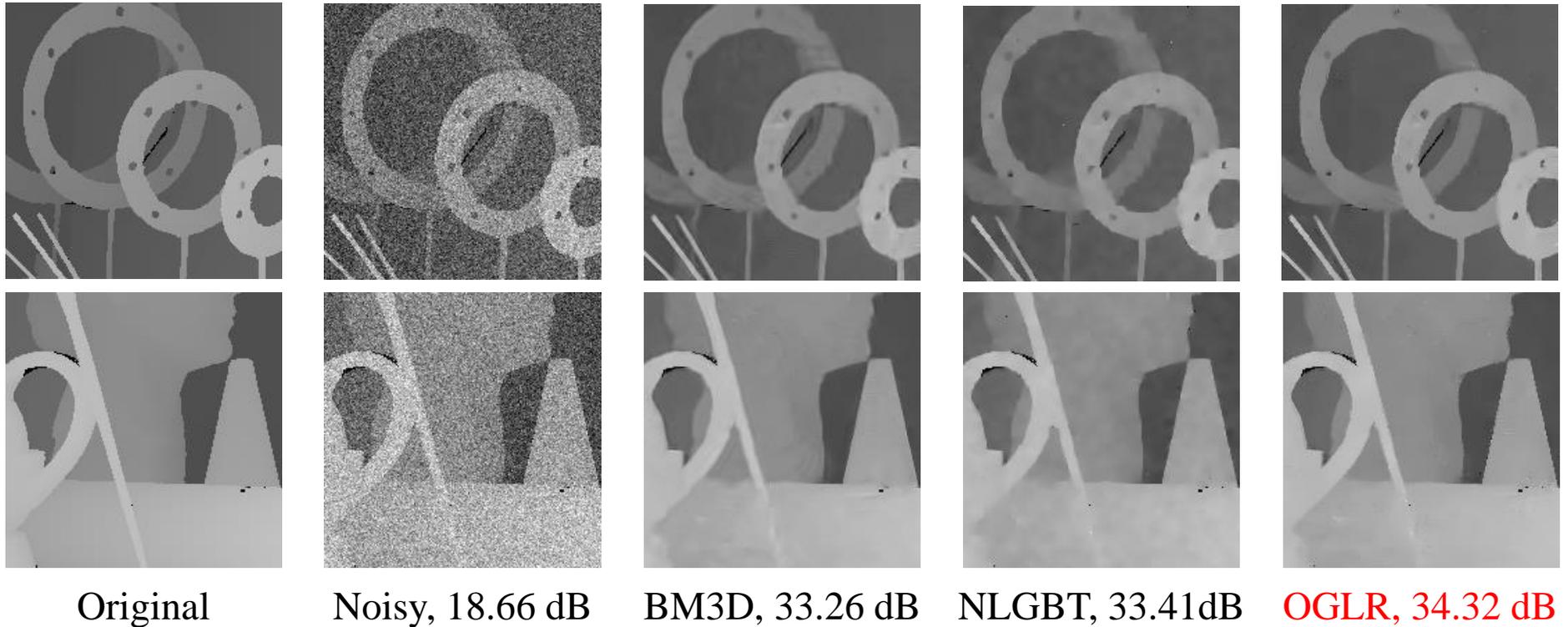
PLOW, 28.11 dB



OGLR, 28.35 dB

Results: depth image denoising

- Subjective comparisons ($\sigma_I = 30$)



GLR for Joint Dequantization / Contrast Enhancement

- Retinex decomposition model:

$$\mathbf{y} = \tau \mathbf{l} \odot \mathbf{r} + \mathbf{z}$$

scalar illumination reflectance noise

- **Objective:** general smoothness for luminance, smoothness w/ negative edges for reflectance.

$$\begin{aligned} \min_{\mathbf{l}, \mathbf{r}} \quad & \mathbf{l}^\top (\mathbf{L}_l + \alpha \mathbf{L}_l^2) \mathbf{l} + \mu \mathbf{r}^\top \mathcal{L}_\tau \mathbf{r} \\ \text{s.t.} \quad & (\mathbf{q} - \frac{1}{2}) \mathbf{Q} \preceq \mathbf{T} \tau \mathbf{l} \odot \mathbf{r} \prec (\mathbf{q} + \frac{1}{2}) \mathbf{Q} \end{aligned}$$

generalized smooth piecewise smooth

- **Constraints:** quantization bin constraints

- **Solution:** Alternating accelerated proximal gradient alg [1].

Results: Contrast Enhancement



(a)

(b)



(c)

(d)



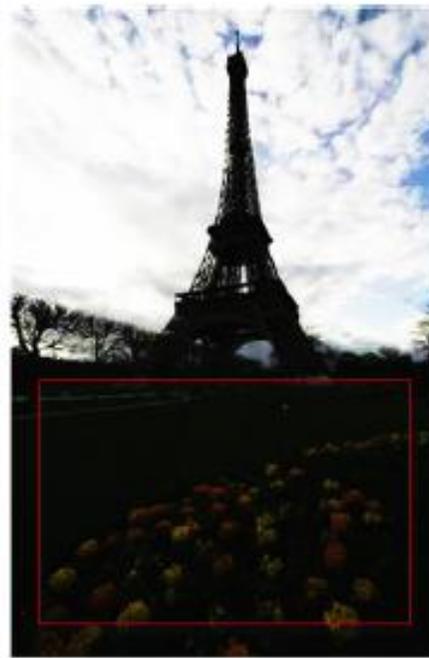
(e)

(f)

Results: Contrast Enhancement



(a)



(b)



(c)



(d)



(e)



(f)

Results: Contrast Enhancement



(a)



(b)



(c)



(d)



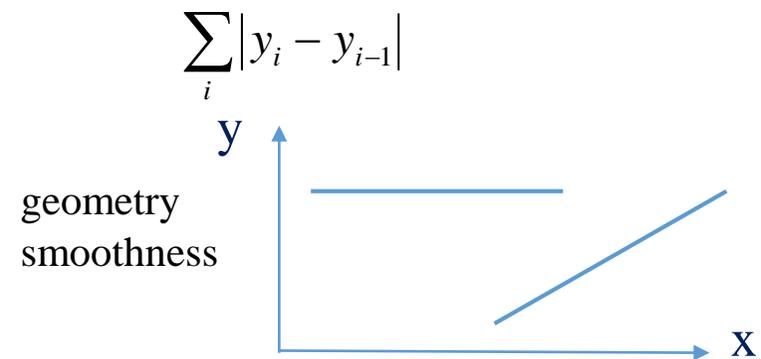
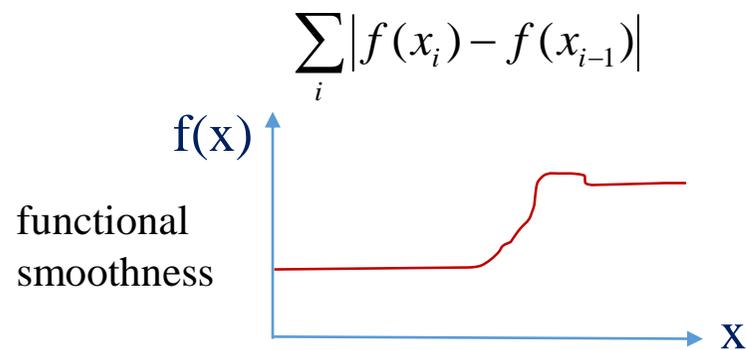
(e)



(f)

GTV for Point Cloud Denoising

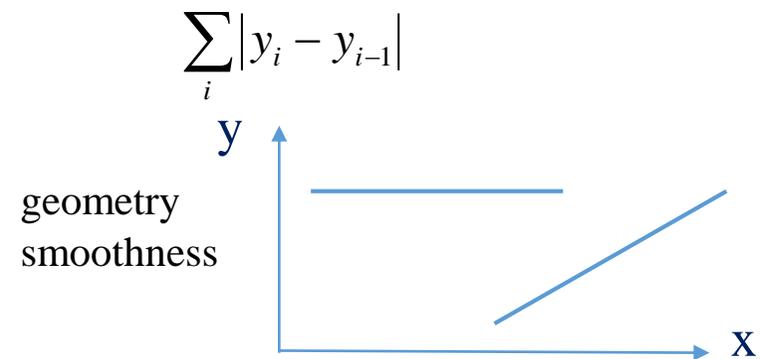
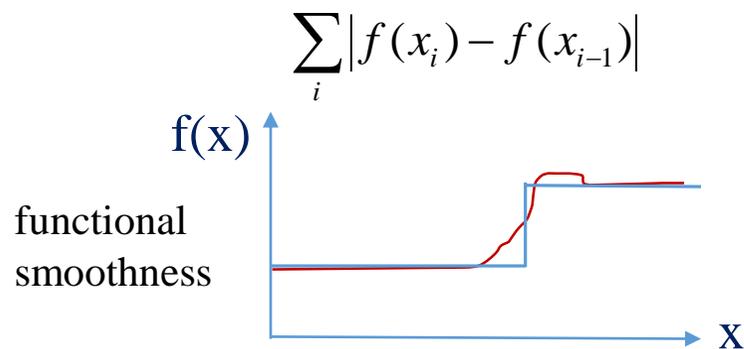
- Acquisition of point cloud introduces noise.
- Point cloud is irregularly sampled 2D manifold in 3D space.
- Not appropriate to apply GTV directly on 3D coordinates [1].
 - only **a singular 3D point has zero GTV value.**



- **Proposal:** Apply GTV is to the surface normals of 3D point cloud—a **generalization of TV to 3D geometry.**

GTV for Point Cloud Denoising

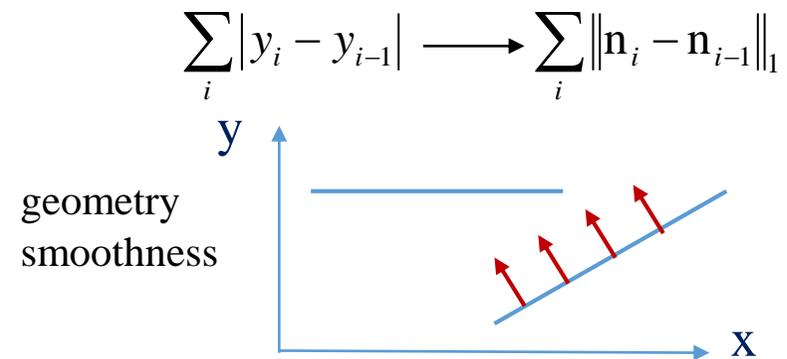
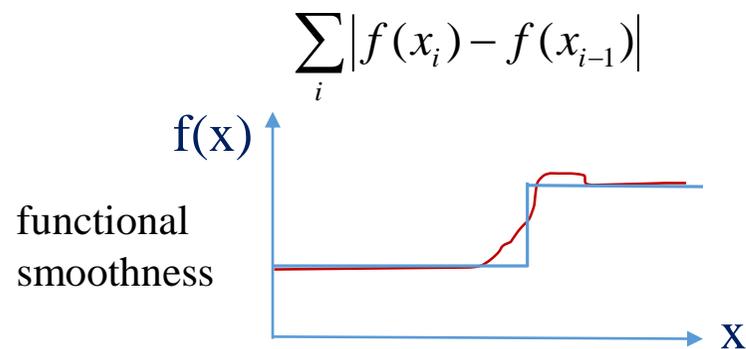
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PC Denoising Algorithm

- Use GTV of surface normals over the K-NN graph:

$$\|\mathbf{n}\|_{\text{GTV}} = \sum_{i,j \in \mathcal{E}} w_{i,j} \|\mathbf{n}_i - \mathbf{n}_j\|_1 \quad \begin{array}{c} \uparrow \mathbf{n}_i \\ \bullet \\ i \end{array} \quad \begin{array}{c} \uparrow \mathbf{n}_j \\ \bullet \\ j \end{array} \quad w_{i,j} = \exp\left(-\frac{\|\mathbf{p}_i - \mathbf{p}_j\|_2^2}{\sigma_p^2}\right)$$

- Denoising problem as l2-norm fidelity plus GTV of surface normals:

$$\min_{\mathbf{p}, \mathbf{n}} \|\mathbf{q} - \mathbf{p}\|_2^2 + \gamma \sum_{i,j \in E} w_{i,j} \|\mathbf{n}_i - \mathbf{n}_j\|_1 \quad \text{smoothness on surface normals}$$

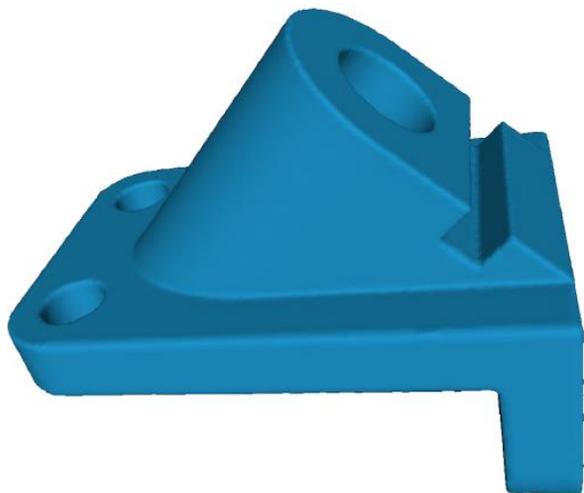
- Surface normal estimation of \mathbf{n}_i is a nonlinear function of \mathbf{p}_i and neighbors.

Proposal:

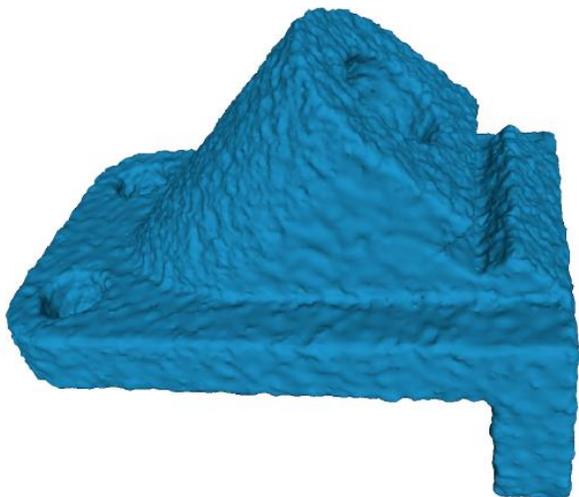
1. Partition point cloud into **two independent classes** (say **red** and **blue**).
2. When computing surface normal for a red node, use only neighboring blue points.
3. Solve convex optimization for red (blue) nodes alternately.

Results: Point Cloud Denoising

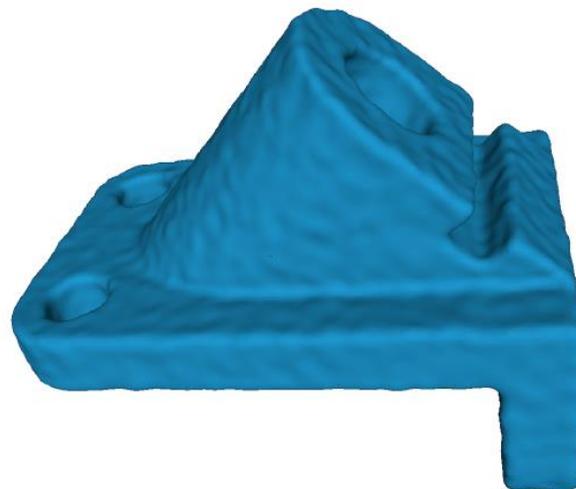
Anchor model ($\sigma=0.3$)



(a) ground truth



(b) noisy input



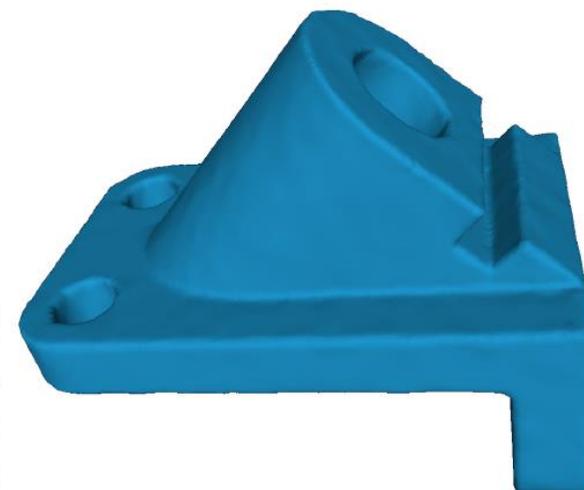
(c) APSS



(d) RIMLS



(e) MRPCA



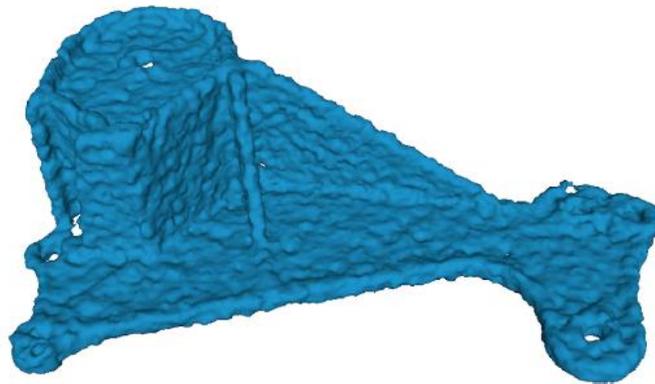
(f) proposed

Results: Point Cloud Denoising

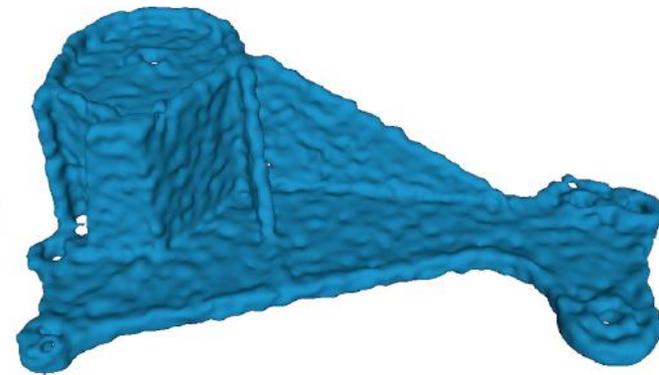
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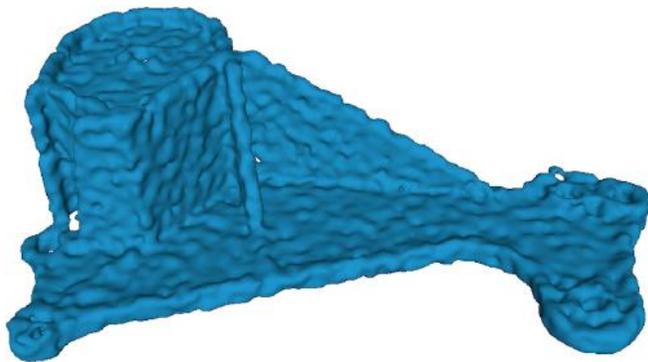
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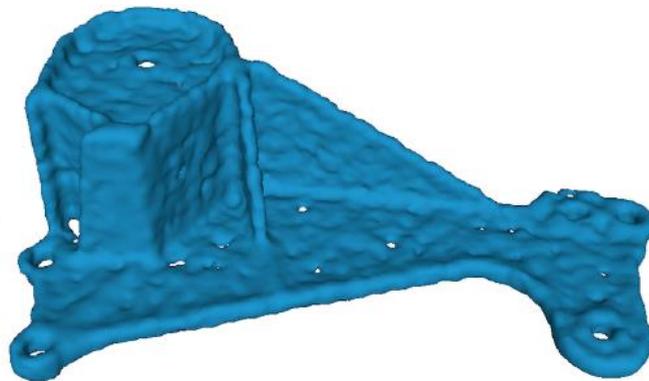
(b) noisy input



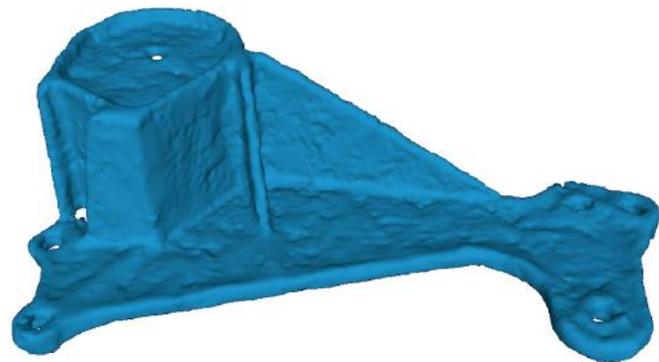
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(f) proposed

PC Super-Res Algorithm

- Add new interior points to low-res point cloud.
 1. Construct triangular mesh using Delaunay triangulation using known points \mathbf{q} .
 2. Insert new points at the centroids of triangles.
- Partition point cloud into **two independent classes** (say **red** and **blue**).
- When computing normal for a red node, use only neighboring blue points.
- Use graph total variation (GTV) of surface normals over the K-NN graph:

smoothness on surface normals

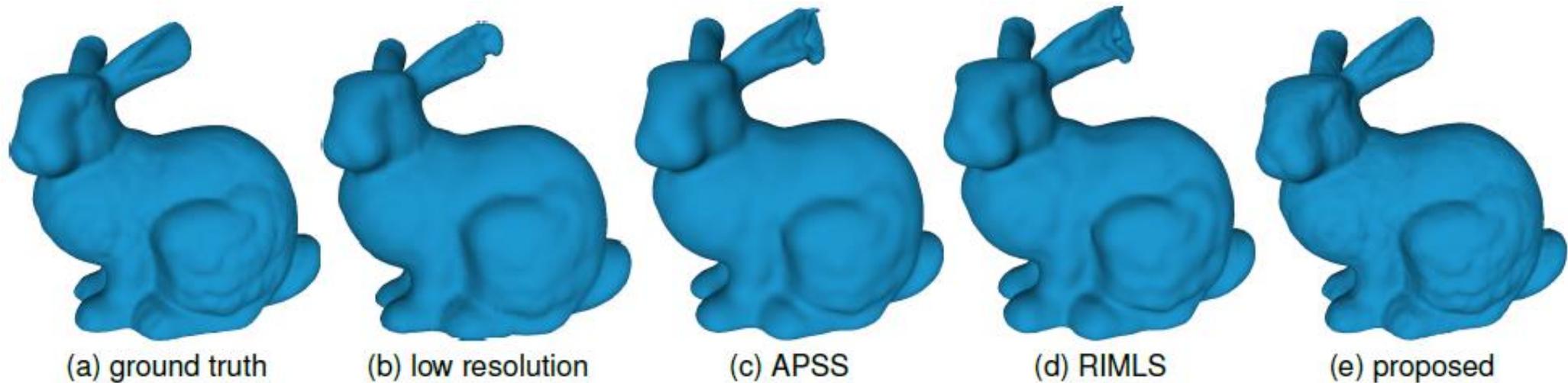
$$\min_{\mathbf{p}, \mathbf{n}} \sum_{i,j \in E} w_{i,j} \|\mathbf{n}_i - \mathbf{n}_j\|_1 \quad \begin{bmatrix} \mathbf{I} & -\mathbf{B} \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{m} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{q} \end{bmatrix}$$

$\mathbf{m}_{i,j} = \mathbf{n}_i - \mathbf{n}_j$

- Solved via augmented Lagrangian + ADMM.

Results: Point Cloud Super-Resolution

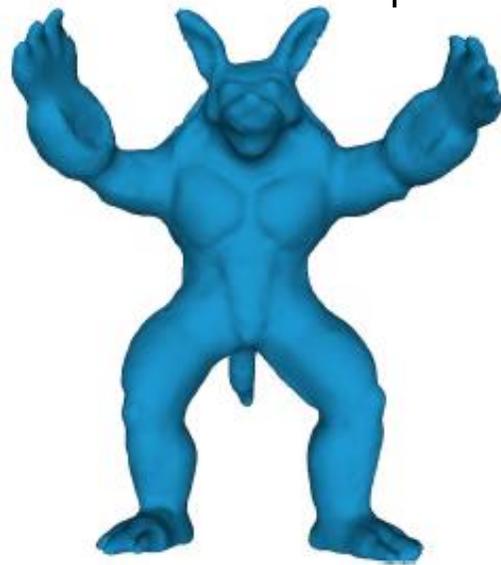
- APSS and RIMLS schemes generate overly smooth models.
- Existing methods result in distorted surfaces with some details lost.



Results: Point Cloud Super-Resolution



(a) ground truth



(b) low resolution



(c) APSS



(d) RIMLS



(e) proposed

Outline

- Defining Graph frequencies
- Inverse Imaging
 - Image denoising
 - Image contrast enhancement
 - 3D point cloud denoising / super-resolution
- Deep GLR
- Semi-Supervised Learning
- Graph Sampling
 - Matrix completion

Unrolling Graph Laplacian Regularizer

- Recall MAP formulation of denoising problem with quadratic graph Laplacian regularizer:

$$\min_x \left\| \mathbf{y} - \mathbf{x} \right\|_2^2 + \mu \mathbf{x}^T \mathbf{L} \mathbf{x}$$

fidelity term  smoothness prior 

- Solution is system of linear equations:

$$(\mathbf{I} + \mu \mathbf{L}) \mathbf{x}^* = \mathbf{y}$$

 linear system of eqn's w/ sparse, symmetric PD matrix

Unrolling Graph Laplacian Regularizer

- Recall MAP formulation of denoising problem with quadratic graph Laplacian regularizer:

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Q: what is the “most appropriate” graph?

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Q: what is the “most appropriate” graph?

Bilateral weights:

$$w_{i,j} = \exp\left(\frac{-\|x_i - x_j\|_2^2}{\sigma_1^2}\right) \exp\left(\frac{-\|l_i - l_j\|_2^2}{\sigma_2^2}\right)$$

Unrolling Graph Laplacian Regularizer

- Deep Graph Laplacian Regularization:**

1. Learn features \mathbf{f} 's using CNN.
2. Compute distance from features.
3. Compute edge weights using Gaussian kernel.
4. Construct graph, solve QP.

$$w_{ij} = \exp\left(-\frac{\text{dist}(i, j)}{2\epsilon^2}\right),$$

$$\text{dist}(i, j) = \sum_{n=1}^N (\mathbf{f}_n(i) - \mathbf{f}_n(j))^2.$$

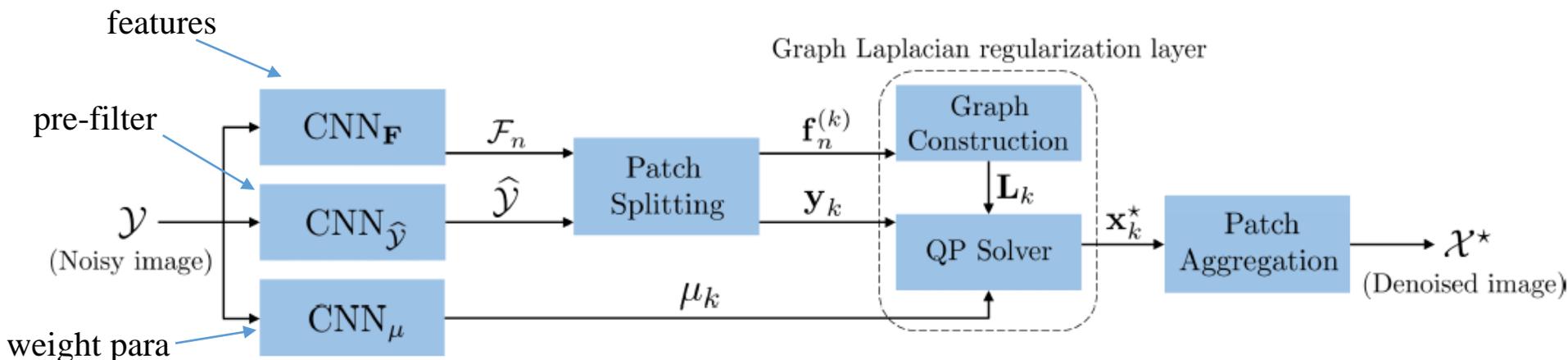


Fig. 1. Block diagram of the proposed GLRNet which employs a graph Laplacian regularization layer for image denoising.

Unrolling Graph Laplacian Regularizer

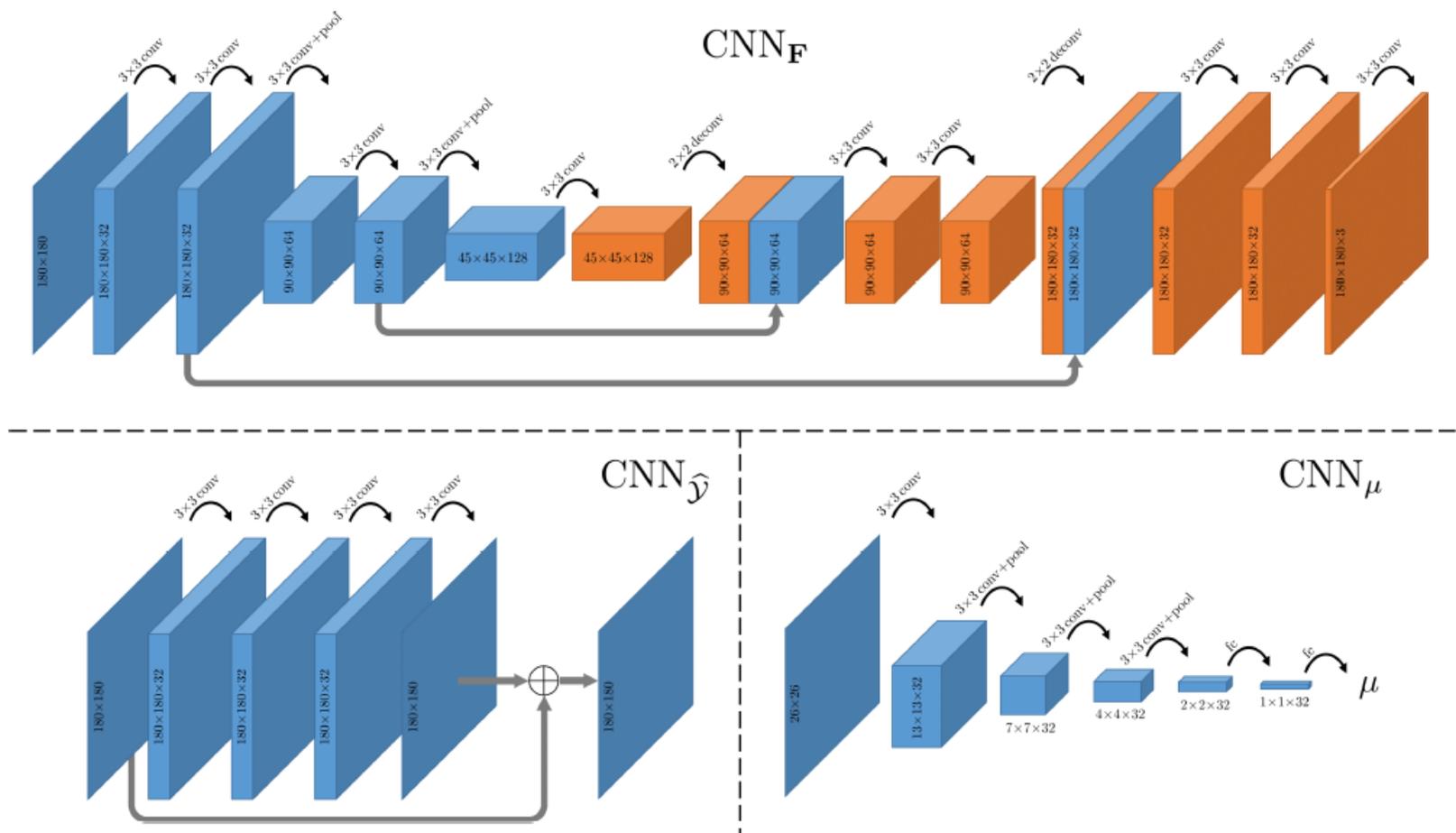


Fig. 3. Network architectures of CNN_F , $\text{CNN}_{\hat{y}}$ and CNN_{μ} in the experiments. Data produced by the decoder of CNN_F is colored in orange.

Unrolling Graph Laplacian Regularizer

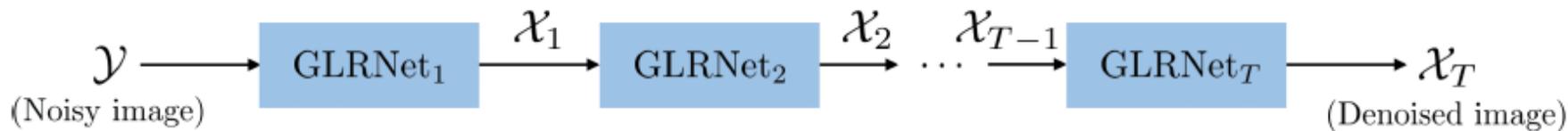


Fig. 2. Block diagram of the overall DeepGLR framework.

- **Graph Model** *guarantees numerical stability of solution:*

$$(\mathbf{I} + \mu \mathbf{L}) \mathbf{x}^* = \mathbf{y}$$

- **Thm 1:** condition number κ of matrix satisfies [1]:

$$\kappa \leq 1 + 2\mu d_{\max},$$

← maximum node degree

- **Observation:** By restricting search space of CNN to degree-bounded graphs, we achieve robust learning.

Experimental Results – Numerical Comparison

- Trained on AWGN on 5 images, patches of size 26-by-26.
- Batch size is 4, model is trained for 200 epochs.
- Trained for both known and blind noise variance.

Table 3. Average PSNR (dB) and SSIM values for Gaussian noise removal.

Noise	Method (PSNR/SSIM)		
	CBM3D	CDnCNN	DeepGLR
15	33.49/ 0.9216	33.80/ 0.9268	33.65/ 0.9259
25	30.68/ 0.8675	31.13/ 0.8799	31.03/ 0.8797
50	27.35/ 0.7627	27.91/ 0.7886	27.86/ 0.7924

Experimental Results – Numerical Comparison

- Cross-domain generalization.
- trained on Gaussian noise, tested on low-light images in (RENOIR).
- Competing methods: DnCNN [1], noise clinic [2].
- outperformed DnCNN by 5.74 dB, and noise clinic by 1.87 dB.

Table 4. Evaluation of cross-domain generalization for real image denoising. The best results are highlighted in boldface.

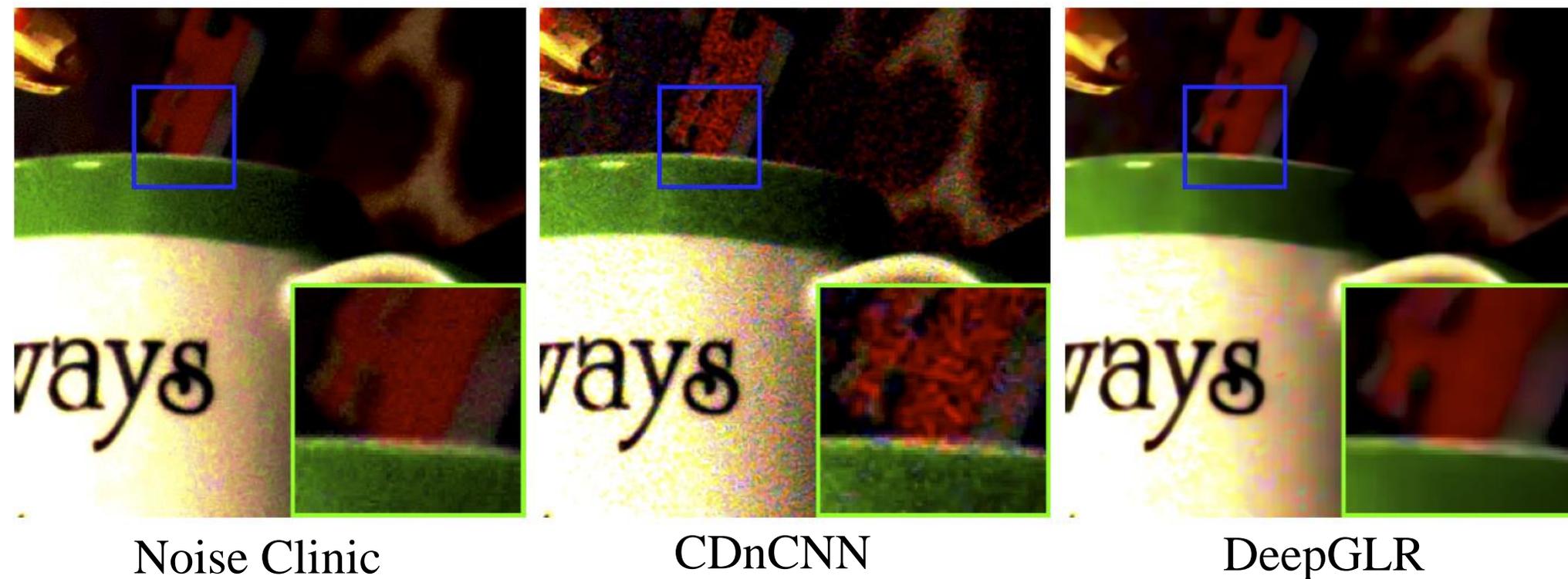
Metric	Noisy	Method		
		Noise Clinic	CDnCNN	DeepGLR
PSNR	20.36	27.43	24.36	30.10
SSIM	0.1823	0.6040	0.5206	0.8028

[1] Kai Zhang et al, “Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising,” *TIP* 2017.

[2] Marc Lebrun et al, “The noise clinic: a blind image denoising algorithm,” *IPOLE* 2015.

Experimental Results – Visual Comparison

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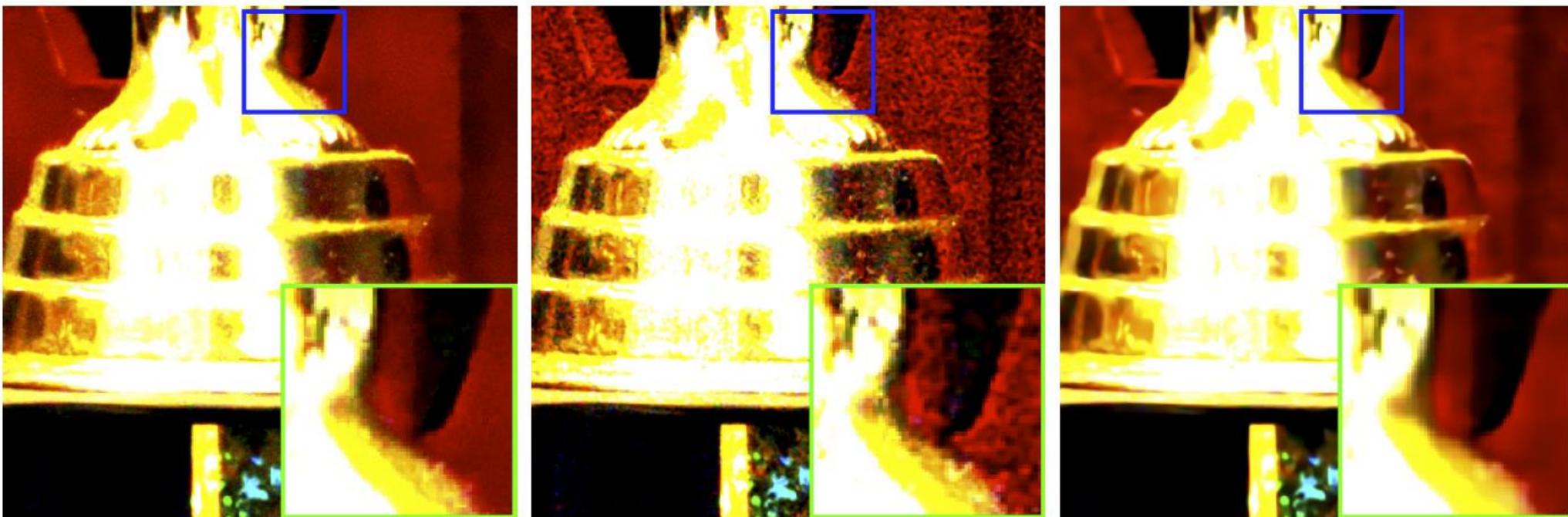


[1] Kai Zhang et al, “Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising,” *TIP* 2017.

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Noise Clinic

CDnCNN

DeepGLR

[1] Kai Zhang et al, “Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising,” *TIP* 2017.

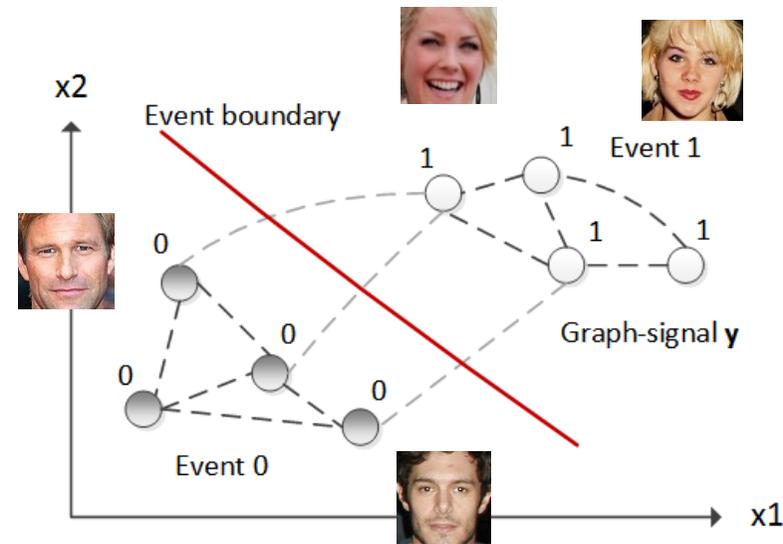
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Semi-Supervised Graph Classifier Learning

- **Binary Classifier:** given feature vector x_i of dimension K , compute $f(x_i) \in \{0,1\}$.
- **Classifier Learning:** given partial, noisy labels (x_i, y_i) , train classifier $f(x_i)$.



example graph-based classifier

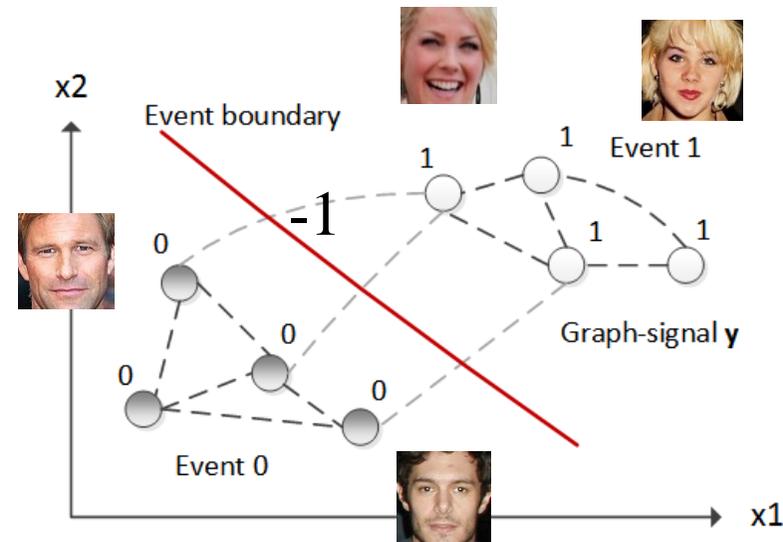
- **GSP Approach** [1]:
 1. Construct **signed similarity graph** with +/- edges.
 2. Pose MAP graph-signal restoration problem.
 3. Perturb graph Laplacian to ensure PSD.
 4. Solve num. stable MAP as sparse lin. system.

[1] Yu Mao, Gene Cheung, Chia-Wen Lin, Yusheng Ji, "Image Classifier Learning from Noisy Labels via Generalized Graph Smoothness Priors," *IEEE IVMSWP Workshop*, Bordeaux, France, July 2016. (**Best student paper award**)

[2] G. Cheung, W.-T. Su, Y. Mao, C.-W. Lin, "Robust Semi-Supervised Graph Classifier Learning with Negative Edge Weights," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 4, no.4, pp.712-726, December 2018.

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w=1

Graph-Signal Smoothness Prior for signed graphs

- Graph Laplacian Regularizer [1]:**

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = \sum_{(i,j) \in \mathcal{E}} w_{i,j} (x_i - x_j)^2 = \sum_k \lambda_k \alpha_k^2$$

GFT coeff

eigenvalues / graph freqs

- Promote large / small inter-node differences depending on edge signs.

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = -1(x_1 - x_2)^2 + (x_2 - x_3)^2$$

Promote large difference

Promote small difference

- Sensible, but numerically unstable.

[1] J. Pang and G. Cheung, "Graph Laplacian regularization for image denoising: Analysis inn the continuous domain," in *IEEE Transactions on Image Processing*, vol. 26, no.4, April 2017, pp. 1770–1785.

Semi-Supervised Learning Formulation

- MAP formulation:

$$\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_0 + \mu \mathbf{x}^T (\mathbf{L} + \Delta) \mathbf{x}$$

l0 fidelity term

perturbation matrix
to ensure PSD!

smoothness prior for signed graph

- One sol'n is $\Delta = \lambda_{\min} \mathbf{I}$, *i.e.* shift all eigenvalues up by $\eta = \lambda_{\min}$.
- **Intuition:** signal variations + signal energies

$$\begin{aligned} \mathbf{x}^T (\mathbf{L} + \Delta) \mathbf{x} &= \mathbf{x}^T \mathbf{L} \mathbf{x} + \eta \mathbf{x}^T \mathbf{I} \mathbf{x} \\ &= \sum_{i,j} w_{i,j} (x_i - x_j)^2 + \eta \sum_i x_i^2 \end{aligned}$$

Results: Semi-Supervised Learning

- Comparisons w/ other classifiers:

TABLE II
CLASSIFICATION ERROR RATES IN THE BANANA DATASET FOR
COMPETING SCHEMES UNDER DIFFERENT TRAINING LABEL ERROR RATES
(THE NUMBERS IN THE PARENTHESES OF THE LAST ROW INDICATE THE
REJECTION RATES)

% label noise	0%	5%	10%	15%	20%
SVM-Linear	54.71%	54.97%	54.70%	53.95%	53.42%
SVM-RBF	12.49%	13.27%	13.72%	16.23%	18.63%
RobustBoost [26]	20.42%	22.73%	24.53%	25.12%	27.52%
Graph-Pos	14.05%	15.89%	18.02%	20.76%	21.93%
Graph-MinNorm	10.23%	12.37%	14.44%	17.41%	18.69%
Graph-Bandlimited [58]	7.53%	11.77%	15.80%	19.14%	21.07%
Graph-AdjSmooth [9]	8.85%	12.08%	15.28%	18.26%	20.67%
Graph-Wavelet [6]	23.18%	24.25%	25.70%	27.15%	30.13%
Proposed-Centroid	5.17%	10.50%	13.79%	16.80%	19.39%
Proposed-Boundary	13.37%	15.68%	18.27%	20.51%	22.72%
Proposed-Hybrid	5.36%	9.43%	12.79%	16.04%	18.43%
Proposed-Rej	3.74% (9.59%)	6.57% (9.89%)	9.26% (9.14%)	12.19% (9.96%)	14.06% (9.95%)

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Results: Semi-Supervised Learning

- Comparisons w/ other classifiers:

TABLE III

**CLASSIFICATION ERROR RATES IN THE FACE GENDER DATASET FOR
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% label noise	0%	5%	10%	15%	20%
SVM-Linear	17.65%	18.22%	18.77%	19.59%	21.6%
SVM-RBF	12.14%	12.16%	12.83%	16.30%	24.01%
RobustBoost [26]	9.15%	11.09%	14.36%	17.36%	20.68%
Graph-Pos	13.15%	13.62%	14.38%	15.39%	16.54%
Graph-MinNorm	7.15%	8.26%	9.48%	10.37%	12.01%
Graph-Bandlimited [58]	5.78%	11.83%	15.30%	19.74%	23.44%
Graph-AdjSmooth [9]	1.25%	5.01%	7.94%	11.45%	15.39%
Graph-Wavelet [6]	20.02%	19.95%	20.12%	20.7%	21.43%
Proposed-Centroid	1.44%	2.96%	4.46%	5.88%	8.07%
Proposed-Boundary	10.81%	12.09%	13.17%	14.33%	15.96%
Proposed-Hybrid	1.71%	3.02%	4.22%	5.75%	7.71%
Proposed-Rej	0.36% (9.70%)	0.68% (9.29%)	1.08% (9.85%)	2.39% (9.08%)	4.18% (9.05%)

Results: Semi-Supervised Learning

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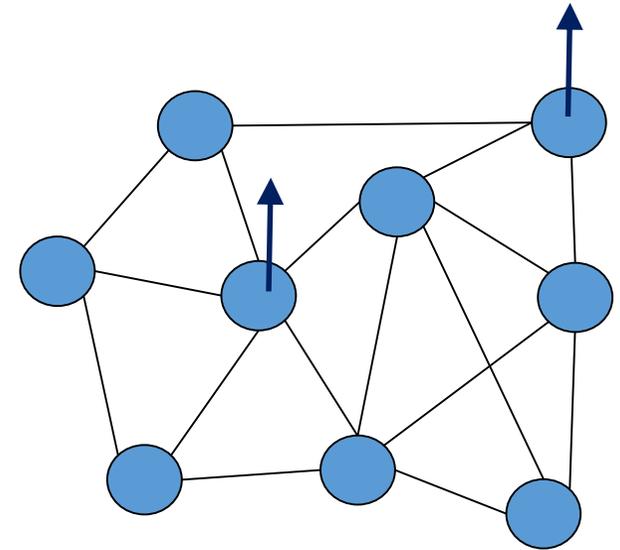
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Outline

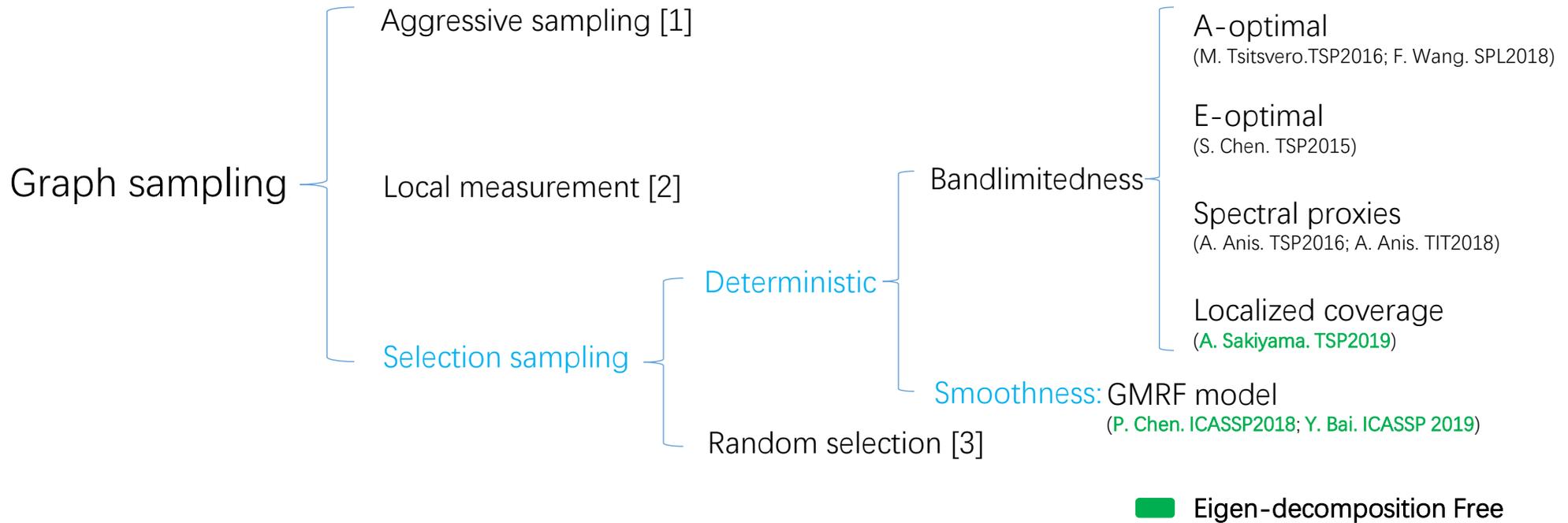
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Graph Sampling (with and without noise)

- **Q:** How to choose best samples for graph-based reconstruction?
- Existing graph sampling strategies extend **Nyquist sampling** to graph data kernels:
 - Assume ***bandlimited*** signal.
 - Greedily select most “informative” samples by computing extreme eigenvectors of sub-matrix.
 - Computation-expensive.



Related Works

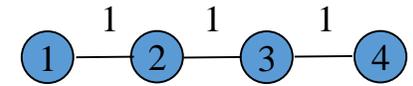


[1] A. G. Marques, S. Segarra, G. Leus, and A. Ribeiro, “**Sampling of graph signals with successive local aggregations.**” *IEEE Transactions on Signal Processing*, vol. 64, no. 7, pp. 1832–1843, 2016.

[2] X. Wang, J. Chen, and Y. Gu, “**Local measurement and reconstruction for noisy bandlimited graph signals,**” *Signal Processing*, vol. 129, pp. 119–129, 2016.

[3] G. Puy, N. Tremblay, R. Gribonval, and P. Vandergheynst, “**Random sampling of bandlimited signals on graphs,**” *Applied and Computational Harmonic Analysis*, vol. 44, no. 2, pp. 446–475, 2018.

Signal Reconstruction using GLR



$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Sample set {2, 4}

- Signal Model:**

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}$$

observation \mathbf{y} , sampling matrix \mathbf{H} , desired signal \mathbf{x} , noise \mathbf{v}

- Signal prior is **graph Laplacian regularizer (GLR)** [1]:

$$\mathbf{x}^T \mathbf{L} \mathbf{x} = \frac{1}{2} \sum_{i,j} w_{i,j} (x_i - x_j)^2 = \sum_k \lambda_k \tilde{x}_k^2$$

signal smooth w.r.t. graph (pointing to $w_{i,j}$), signal contains mostly low graph freq. (pointing to \tilde{x}_k^2)

- MAP Formulation:**

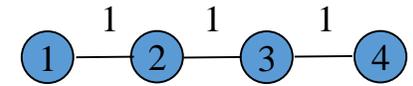
$$\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 + \mu \mathbf{x}^T \mathbf{L} \mathbf{x}$$

fidelity term (pointing to $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2$), signal prior (pointing to $\mu \mathbf{x}^T \mathbf{L} \mathbf{x}$)

$$(\mathbf{H}^T \mathbf{H} + \mu \mathbf{L}) \mathbf{x}^* = \mathbf{y}$$

linear system of eqn's solved using *conjugate gradient*

Stability of Linear System



- Examine system of linear equations :

$$(\mathbf{H}^T \mathbf{H} + \mu \mathbf{L}) \mathbf{x}^* = \mathbf{y}$$

coefficient matrix \mathbf{B}

- Stability depends on the **condition number** ($\lambda_{\max} / \lambda_{\min}$) of coeff. matrix \mathbf{B} .

- λ_{\max} is upper-bounded by $1 + \mu 2^* d_{\max}$.

- Goal:** select samples to maximize λ_{\min} (without computing eigen-pairs)!

- Also minimizes worst-case MSE:

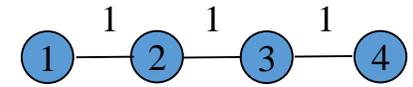
$$\|\hat{\mathbf{x}} - \mathbf{x}\|_2 \leq \mu \left\| \frac{1}{\lambda_{\min}(\mathbf{B})} \right\|_2 \|\mathbf{L}(\mathbf{x} + \tilde{\mathbf{n}})\|_2 + \|\tilde{\mathbf{n}}\|_2$$

$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

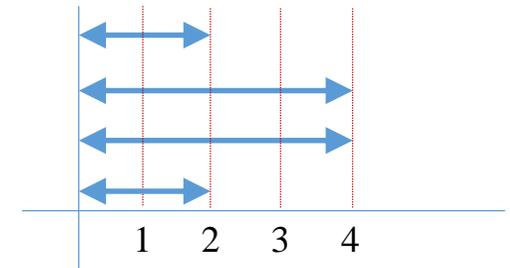
$$\mathbf{H}^T \mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Sample set {2, 4}

Gershgorin Circle Theorem



$$\mathbf{L} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



- **Gershgorin Circle Theorem:**

- Row i of \mathbf{L} maps to a **Gershgorin disc** w/ centre L_{ii} and radius R_i

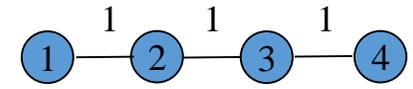
$$R_i = \sum_{j \neq i} |L_{ij}|$$

- λ_{\min} is lower-bounded by smallest left-ends of Gershgorin discs:

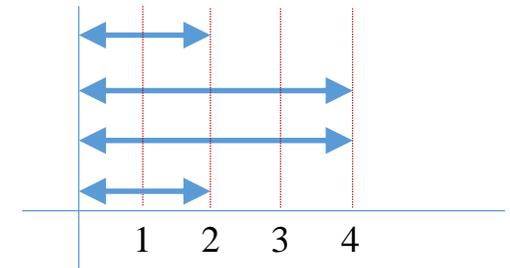
$$\min_i L_{i,i} - R_i \leq \lambda_{\min}$$

- Graph Laplacian \mathbf{L} has all Gershgorin disc left-ends at 0 $\rightarrow \mathbf{L}$ is psd.

Gershgorin Disc Alignment (math)



$$\mathbf{B} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



Sample set { }

Scale factor {1,1,1,1}

- **Main Idea:** Select samples to max smallest disc left-end of coefficient matrix \mathbf{B} :

$$\mathbf{B} = \mathbf{H}^T \mathbf{H} + \mu \mathbf{L} \quad \leftarrow \text{coeff. matrix}$$

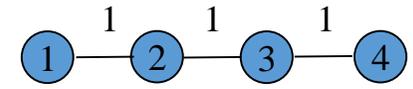
- Sample node \rightarrow shift disc.
- Consider similar transform of \mathbf{B} :

$$\mathbf{C} = \mathbf{S} \mathbf{B} \mathbf{S}^{-1} \quad \leftarrow \text{similarity transform}$$

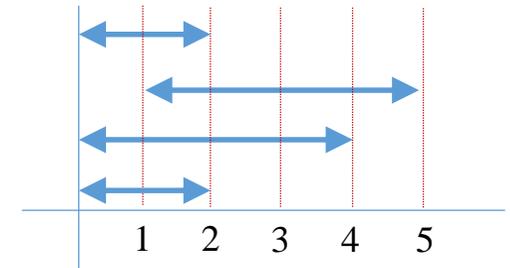
\swarrow
diagonal matrix w/ scale factors

- Scale row \rightarrow **expand** disc radius.
 \rightarrow **shrink** neighbors' disc radius.

Gershgorin Disc Alignment (math)



$$\mathbf{B} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



Sample set {2}

Scale factor {1,1,1,1}

- **Main Idea:** Select samples to max smallest disc left-end of coefficient matrix \mathbf{B} :

$$\mathbf{B} = \mathbf{H}^T \mathbf{H} + \mu \mathbf{L} \quad \leftarrow \text{coeff. matrix}$$

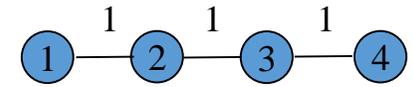
- Sample node \rightarrow shift disc.
- Consider similar transform of \mathbf{B} :

$$\mathbf{C} = \mathbf{S} \mathbf{B} \mathbf{S}^{-1} \quad \leftarrow \text{similarity transform}$$

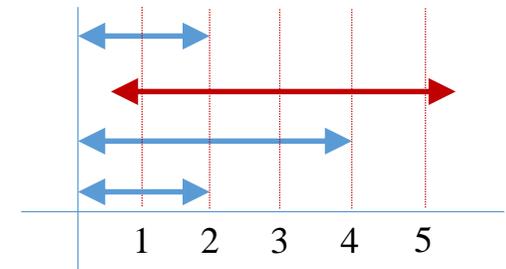
\swarrow
diagonal matrix w/ scale factors

- Scale row \rightarrow **expand** disc radius.
 \rightarrow **shrink** neighbors' disc radius.

Gershgorin Disc Alignment (math)



$$\mathbf{B} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



Sample set {2}

Scale factor {1, s_2 , 1, 1}

- **Main Idea:** Select samples to max smallest disc left-end of coefficient matrix \mathbf{B} :

$$\mathbf{B} = \mathbf{H}^T \mathbf{H} + \mu \mathbf{L} \quad \leftarrow \text{coeff. matrix}$$

- Sample node \rightarrow shift disc.
- Consider similar transform of \mathbf{B} :

$$\mathbf{C} = \mathbf{S} \mathbf{B} \mathbf{S}^{-1} \quad \leftarrow \text{similarity transform}$$

\swarrow
 diagonal matrix w/ scale factors

- Scale row \rightarrow **expand** disc radius.
 \rightarrow **shrink** neighbors' disc radius.

Gershgorin Disc Alignment (math)

- **Main Idea:** Select samples to max smallest disc left-end of coefficient matrix **B**:

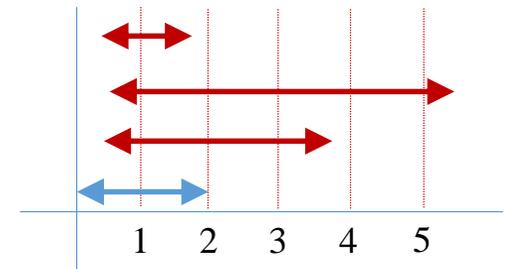
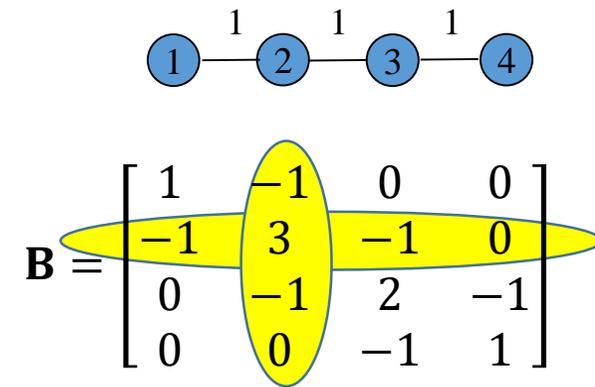
$$\mathbf{B} = \mathbf{H}^T \mathbf{H} + \mu \mathbf{L} \quad \leftarrow \text{coeff. matrix}$$

- Sample node \rightarrow shift disc.
- Consider similar transform of **B**:

$$\mathbf{C} = \mathbf{S} \mathbf{B} \mathbf{S}^{-1} \quad \leftarrow \text{similarity transform}$$

\swarrow
 diagonal matrix w/ scale factors

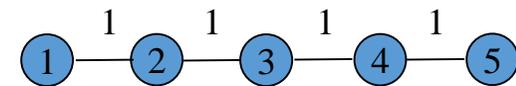
- Scale row \rightarrow **expand** disc radius.
 \rightarrow **shrink** neighbors' disc radius.



Sample set {2}

Scale factor {1, s_2 , 1, 1}

Gershgorin Disc Alignment (math)



Breadth First Iterative Sampling:

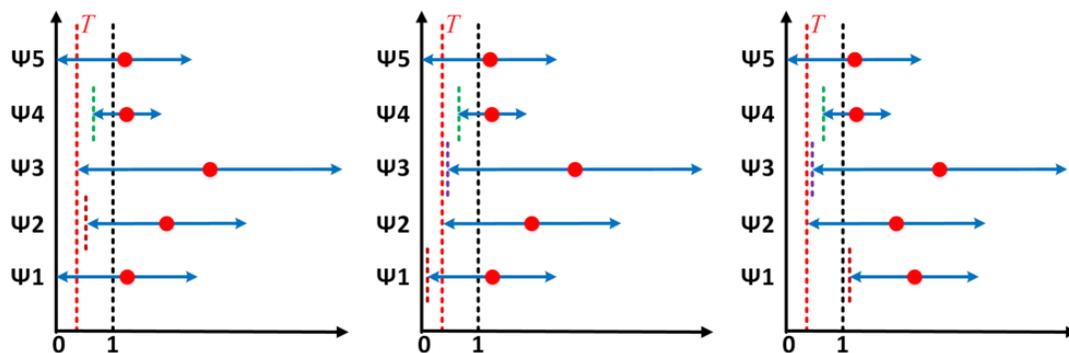
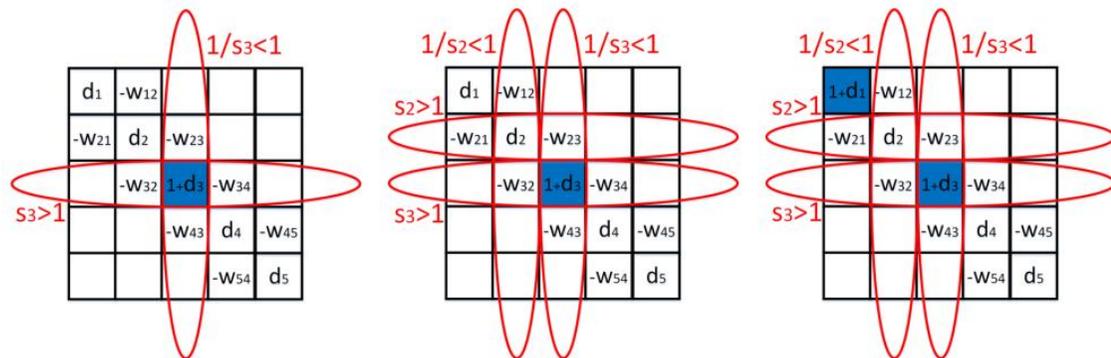
- Given initial node set, threshold T .

- Sample chosen node i
(shift disc)
- Scale row i
(expand disc radius i to T)
- If disc left-end of connected node $j > T$,
Scale row j
(expand disc radius j to T)

Else,

Add node j to node set.

- Goto 1 if node set not empty.
- Output sample set and count K .



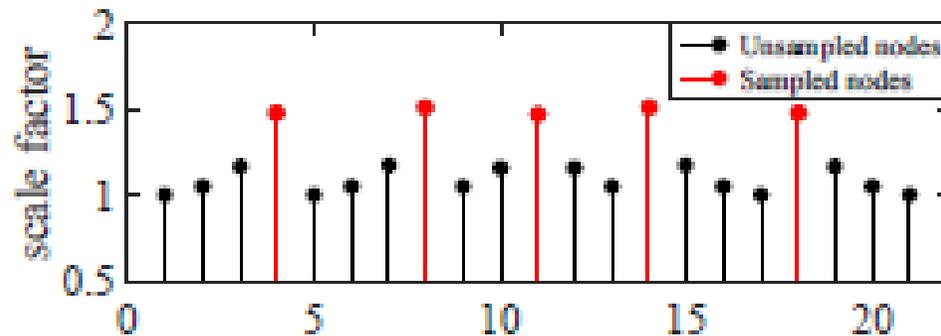
Gershgorin Disc Alignment (math)

- **Binary Search with BFIS:**

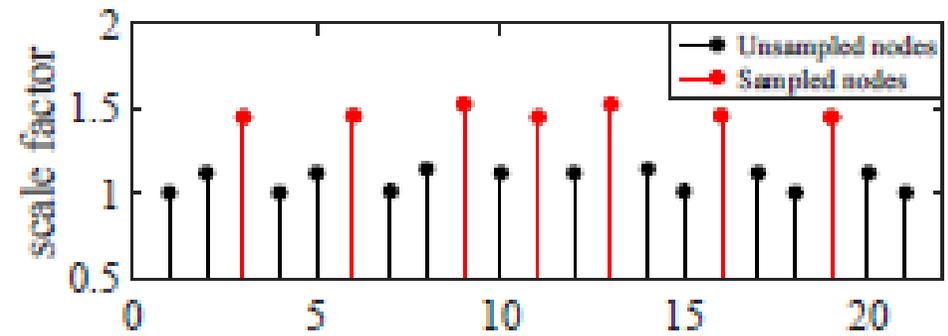
- Sample count K inverse proportional to threshold T .
- Binary search on T to drive count K to budget.

- **Example:** line graph with equal edge weight.

- Uniform sampling.



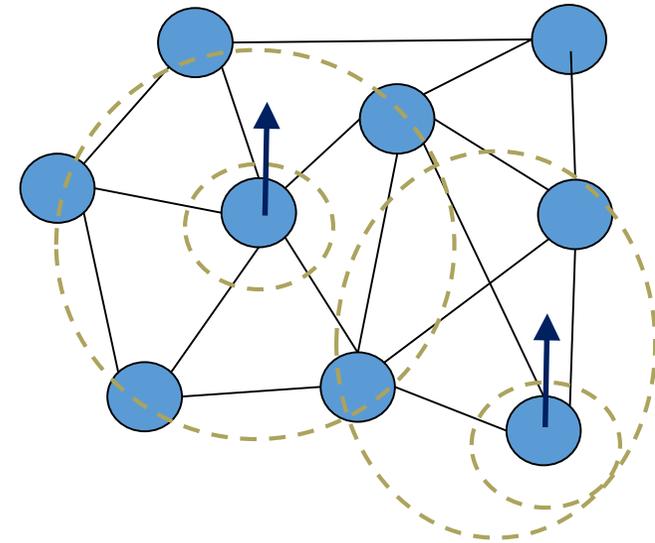
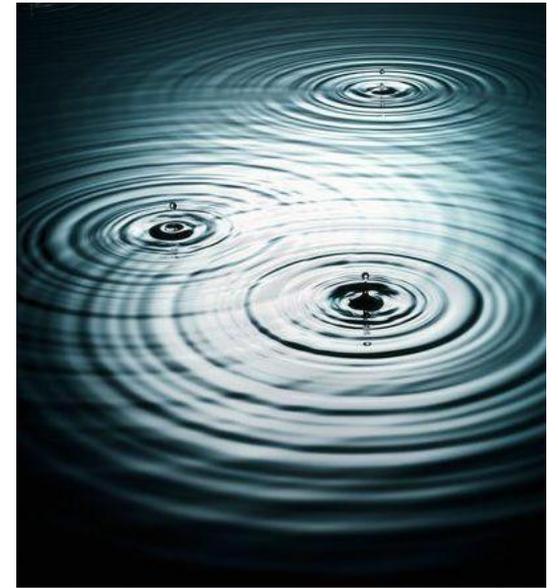
(a)



(b)

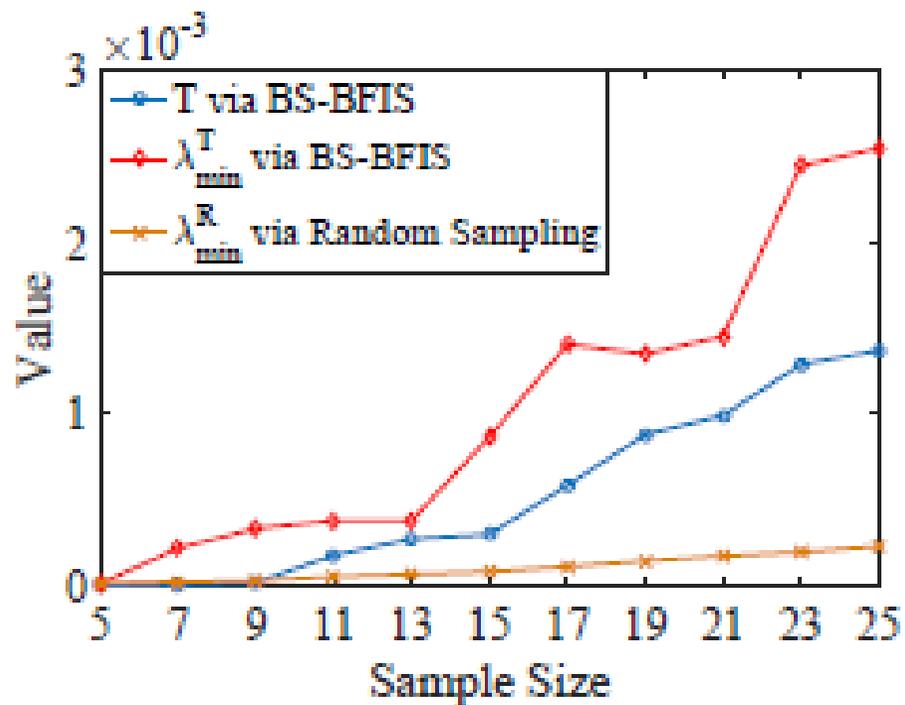
Disc-based Sampling (intuition)

- **Analogy:** throw pebbles into a pond.
- **Disc Shifting:** throw pebble at sample node i .
- **Disc Scaling:** ripple to neighbors of node i .
- **Goal:** Choose samples to maximize the ripple on the least disturbed node.

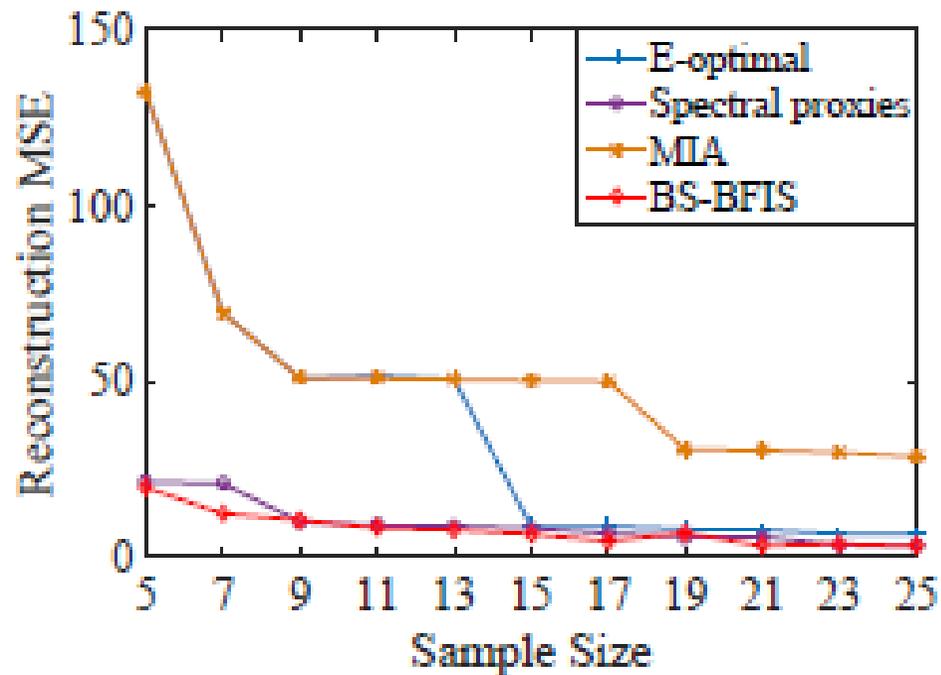


Results: Graph Sampling

- GDA has larger λ_{\min} and smaller condition number.
- GDA has smaller reconstruction MSE.
- GDA has 10% of computation cost of Spectral proxies.



(a)



(b)

Results: Graph Sampling

- Journal version [1]:
 1. formalize the max λ_{\min} GDA optimization problem,
 2. derive linear-time heuristic based on **set cover** approx. alg.

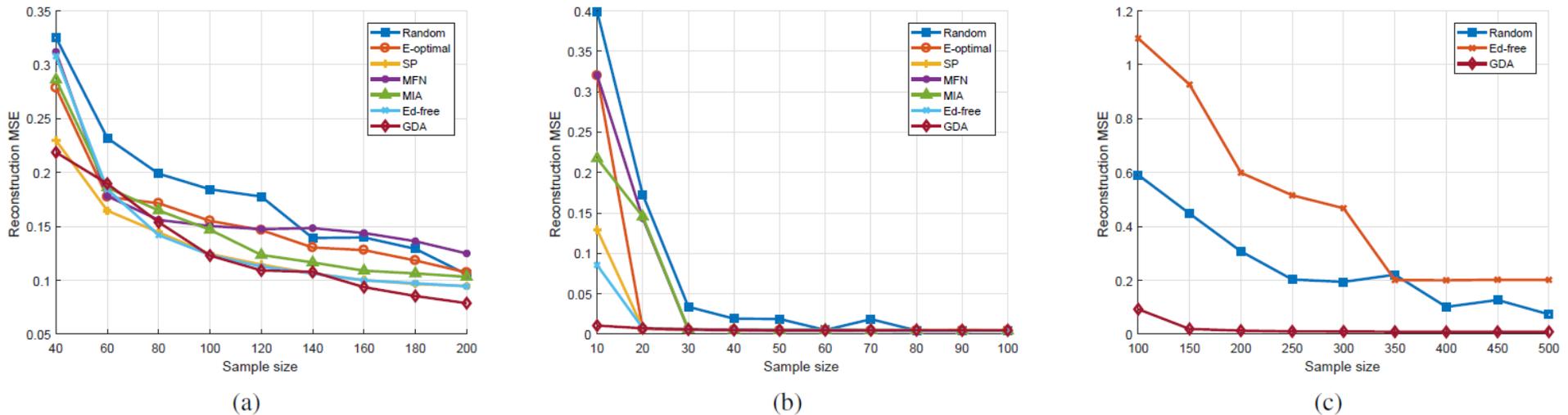


Fig. 5. Reconstruction results for different graphs and sampling algorithms. (a) Random sensor graph of size 1000. (b) Community graph of size 500 with 10 random communities. (c) Minnesota road graph of size 2642..

- **Small graphs:** GDA has roughly the same reconstruction MSE.
- **Large graphs:** GDA has smallest reconstruction MSE.

Matrix Completion

- Fill in missing entries in a matrix:
(Low-rank matrix recovery problem)

$$\min_{\mathbf{X} \in \mathbb{R}^{M \times N}} \text{rank}(\mathbf{X})$$

$$\text{s.t. } X_{i,j} = M_{i,j}, \quad \forall i, j \in S$$

1	2			
	4			
		3		
				2
			3	

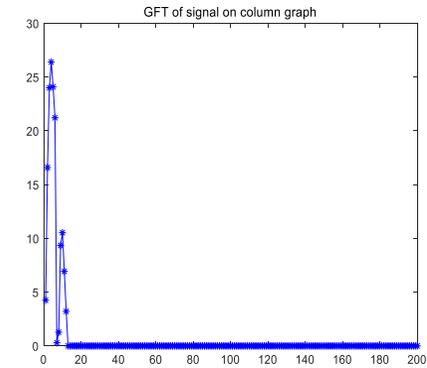
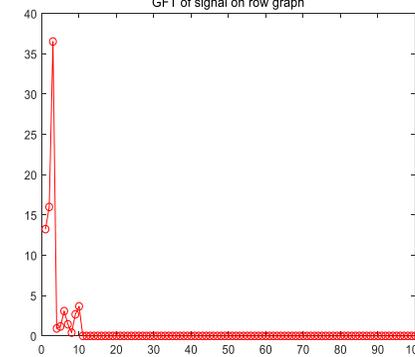
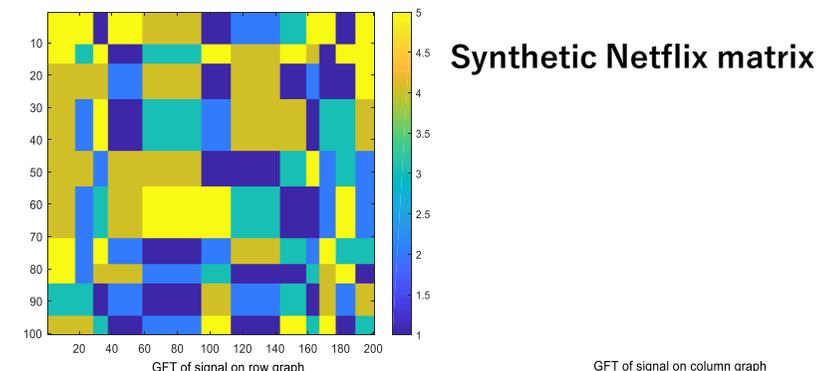
- Examples of applications:
 - **Recommendation system**—making rating prediction.
 - **Remote sensing**—infer full covariance matrix from partial correlations.
 - **Structure-from-motion** in computer vision.

Matrix Completion

- Convex relaxation to **nuclear norm**:

$$\min_{\mathbf{X} \in \mathbb{R}^{M \times N}} \|\mathbf{X}\|_*$$

$$s.t. \quad X_{i,j} = M_{i,j}, \quad \forall i, j \in S$$



Graph Fourier transform on L_r Graph Fourier transform on L_c

- Proximal Gradient**: SVD plus singular value soft-thresholding.
- Use **dual graph-signal smoothness prior** to promote low rank [1]:

$$\min_{\mathbf{X} \in \mathbb{R}^{M \times N}} \text{tr}(\mathbf{X}^T \mathbf{L}_r \mathbf{X}) + \gamma \text{tr}(\mathbf{X} \mathbf{L}_c \mathbf{X}^T) + \mu \|\mathbf{S} \circ \mathbf{M} - \mathbf{S} \circ \mathbf{X}\|_F^2$$

- Unconstrained convex objective solvable via ADMM, conjugate gradient.

Results: Sampling for matrix completion

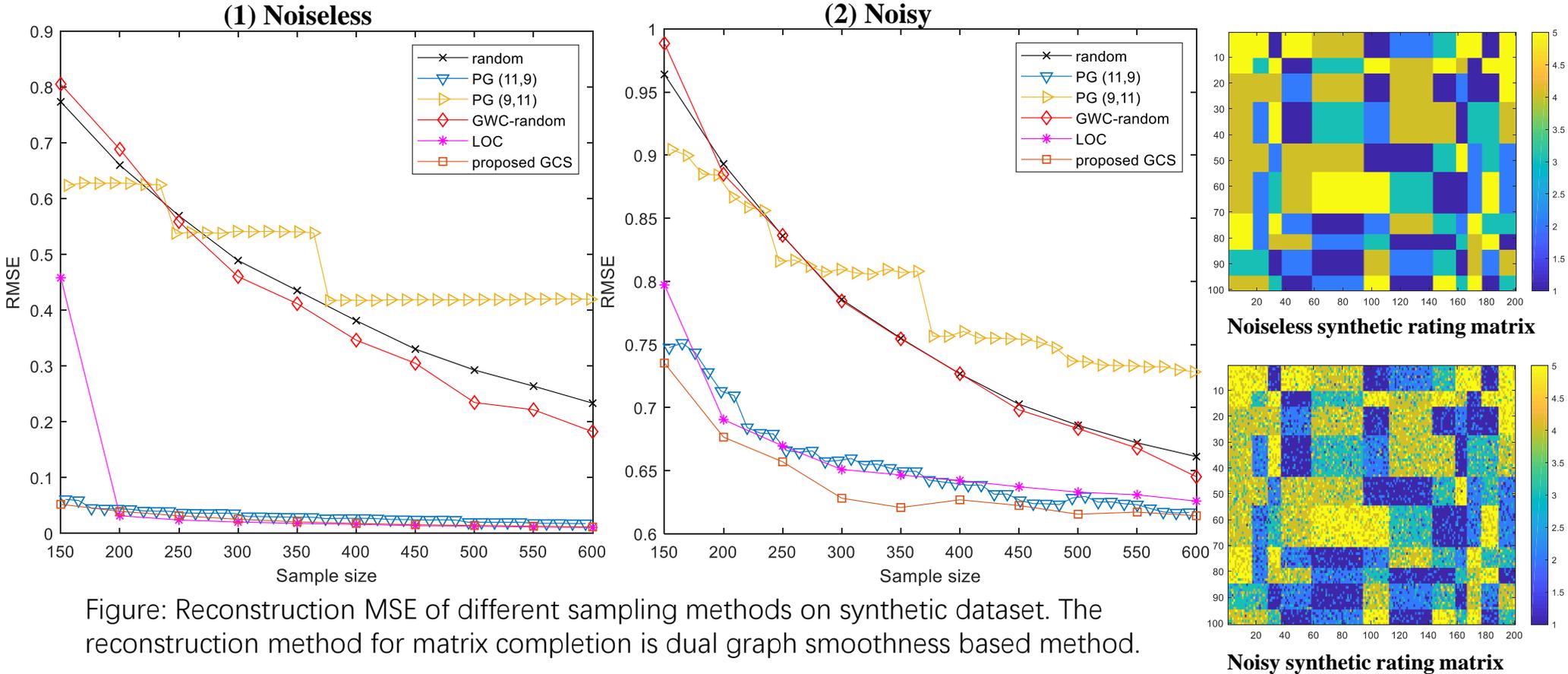


Figure: Reconstruction MSE of different sampling methods on synthetic dataset. The reconstruction method for matrix completion is dual graph smoothness based method.

Comparison methods: PG [1]; GWC-random [2]; LOC [3]

- [1] Guillermo Ortiz-Jiménez, Mario Coutino, Sundeep Prabhakar Chepuri, and Geert Leus. “Sampling and reconstruction of signals on product graphs”. arXiv preprint arXiv:1807.00145, 2018.
- [2] G. Puy, N. Tremblay, R. Gribonval, and P. Vandergheynst, “Random sampling of bandlimited signals on graphs,” Applied and Computational Harmonic Analysis, vol. 44, no. 2, pp. 446–475, 2018.
- [3] A. Sakiyama, Y. Tanaka, T. Tanaka, and A. Ortega, “Eigendecomposition-free sampling set selection for graph signals,” IEEE Transactions on Signal Processing, 2019.

Results: Sampling for matrix completion

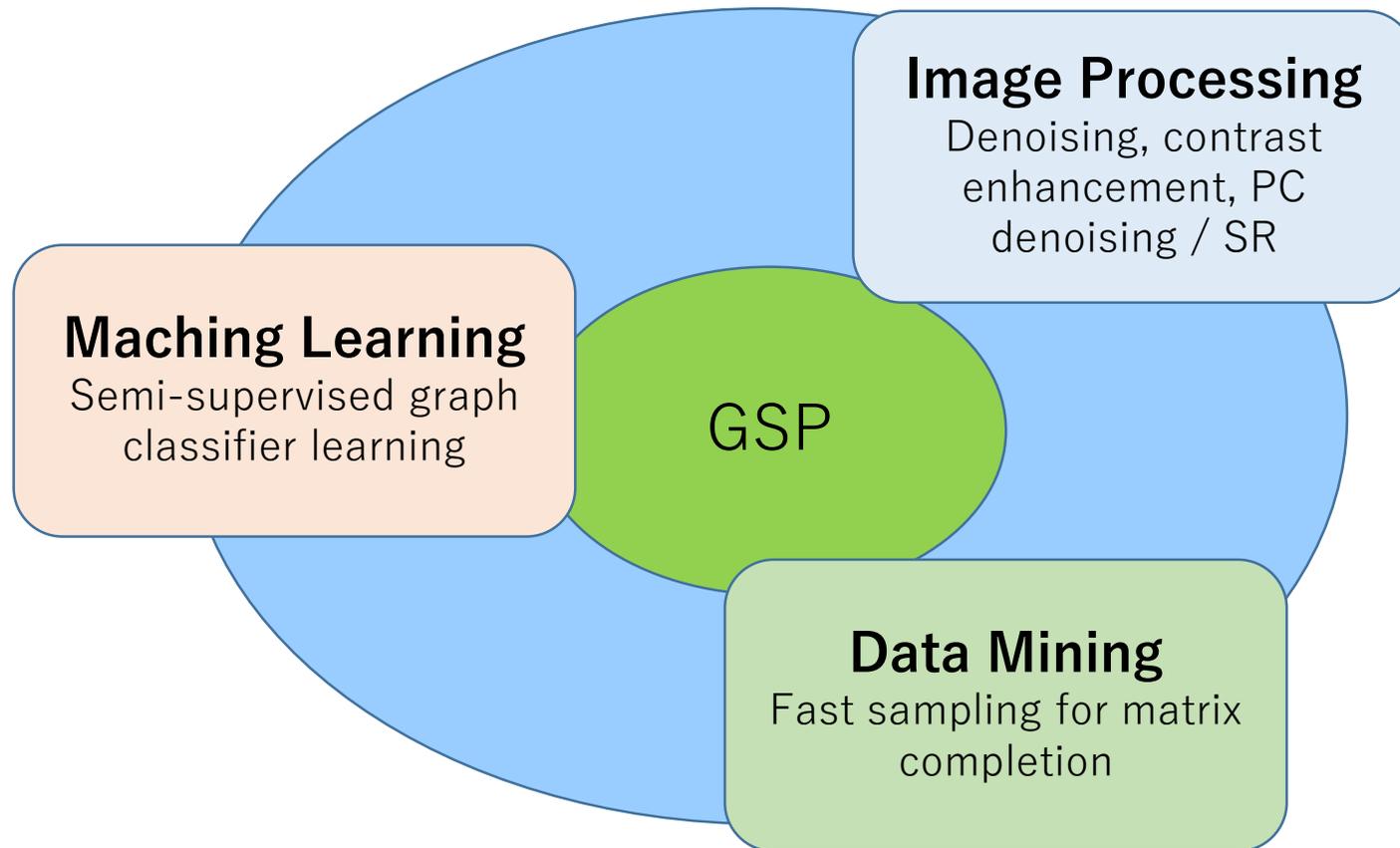
	SVT [1]	GRALS [2]	GMC [3]	NMC [4]
G1	1.021 1.031	0.947 0.931	1.036 1.037	0.890 0.888
G2	1.021 0.983	0.945 0.893	1.118 1.054	0.890 0.858

Table: RMSE of different matrix completion methods on **Mocielens_100k** dataset with different sampling strategies on **Feature-based graph** (G1) and **Content-based graph** (G2). In each grid, the value on left side belongs to random sampling; the right side value is of *our proposed IGCS sampling*. The best performance in each row is marked in bold and red. In our experiments, the sampling budget is 80k out of 100k available ratings; We first use random 60k samples as given, and then proceed to sample the next 20k samples base on random sampling or the proposed IGCS sampling.

- [1] J. Cai, E. J. Candes, and Z. Shen. “A singular value thresholding algorithm for matrix completion”. preprint, 2008.
- [2] N. Rao, H.-F. Yu, P. K. Ravikumar, and I. S. Dhillon. “Collaborative filtering with graph information: Consistency and scalable methods”. In Proc. NIPS, 2015.
- [3] V. Kalofolias, X. Bresson, M. M. Bronstein, and P. Vandergheynst. “Matrix completion on graphs.” 2014.
- [4] D. M. Nguyen, E. Tsiligianni, and N. Deligiannis, “Extendable neural matrix completion,” in Proc. IEEE Int. Conf. Acoust., Speech Signal Process., 2018, pp. 1–5.

Summary

- Graph Spectral Analysis Tools
 - Similarity graph, graph frequencies.



Q&A

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- Homepage: <https://www.eecs.yorku.ca/~genec/index.html>

GSP and Graph-related Research

GSP: SP framework that unifies concepts from multiple fields.

