JOINT DENOISING AND CONTRAST ENHANCEMENT OF IMAGES USING GRAPH LAPLACIAN OPERATOR

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ABSTRACT

Images and videos are often captured in poor light conditions, resulting in low-contrast images that are corrupted by acquisition noise. To recreate a high-quality image for visual observation, the captured image must be denoised and contrast-enhanced. Conventional methods perform these two tasks in two separate stages: an image is first denoised, followed by an enhancement procedure. In this paper, we propose to jointly denoise and enhance an image in one unified optimization framework. The crux of the optimization rests on the definition of the enhancement operator, described by a graph Laplacian matrix $\mathbf{H}$. The operator must enhance the high frequency details of the original image without amplifying additive noise. We propose a graph-based low-pass filtering approach to denoise edge weights in the graph, resulting in a more robust estimate of $\mathbf{H}$. Experimental results show that our proposed joint approach can outperform the separate approach in demonstrable image quality.

***Index Terms*—** image restoration, graph signal processing

1. INTRODUCTION

Images and videos are often forced to be taken under non-ideal lighting conditions. For example, surveillance cameras capture videos in poorly lit offices and hallways at night because it is not economical to install permanent lighting. Another example is outdoor nighttime animal watching, where excessive illumination would disturb the animals' natural nocturnal environments. While advanced image sensors[1] can now capture good quality images even in dark environments, these high-end cameras tend to be very expensive. For the majority of cameras then, one need to rely on post image processing techniques to convert poorly lit images to high-quality images pleasant for human observation. We focus on this image transformation problem in the paper.

Technically, an image captured in low lighting suffers from two shortcomings. First, few number of photons aggregating on a pixel square means the acquired signal suffers from noise due to low Signal-to-Noise Ratio (SNR). Second, low signal strength also means insufficient luminance contrast; the high-frequency details are too weak to be observable by the human eye. The required post-processing thus needs to perform both denoising and contrast enhancement. Naïvely, one can perform the two tasks separately: first a denoising method (*e.g.* bilateral filter [2]) is applied to remove visible noise, then a separate contrast enhancement procedure (*e.g.* histogram-based OCTM [3]) is employed to amplify details in the image. We argue that such separate approach is sub-optimal in general. Denoising is a tradeoff between smoothing (to remove high frequency components due to additive noise) and detail preservation. Thus, a typical denoised image has over-smoothed spatial regions, leaving little details for the contrast enhancement engine to amplify.

In this paper, we present instead a joint denoising and contrast enhancement optimization framework to accomplish both tasks simultaneously. The crux of our proposal is the design of the enhancement operator: while original image should be suitably amplified, undesirable noise should instead be attenuated. Leveraging on recent advance in graph signal processing (GSP) [4], we propose to use the normalized graph Laplacian as the contrast enhancement operator. We propose a graph-based low-pass filtering approach to denoise edge weights in the graph that are estimated from observed noisy pixels, resulting in a more robust enhancement operator. *To the best of our knowledge, we are the first in the literature to jointly denoise and contrast-enhance poorly lit images.* Experimental results show that our proposed joint approach can outperform the conventional separate approach in demonstrable subjective image quality.

The outline of the paper is as follows. We first review related work in Section 2. We describe our problem formulation and corresponding optimization procedures in Section 3 and 4, respectively. Finally, experimentation and conclusions are presented in Section 5 and 6.

2. RELATED WORK

Image denoising is the simplest inverse imaging problem and has been studied extensively in the literature. Popular methods including *bilateral filter* (BF) [2], *nonlocal means* (NLM) [5], and sparse coding [6]. While our work also assumes

[1] A recent *Science* article describes a new image sensor requiring very few photon counts [1].

sparse representation of an image patch (each signal can be represented by a sparse linear combination of learned dictionary atoms), we formulate a joint denoising and contrast enhancement problem and fulfill both tasks simultaneously.

Contrast enhancement has become an active research topic in image processing. One widely used method manipulates the histogram of the input image to separate the gray levels of higher probability further apart from the neighboring gray levels. Representative works include the well-known histogram equalization and its variant [7, 3]. Wu [3] presented a more rigorous study of histogram-based contrast enhancement method, and proposed optimal contrast-tone apping (OCTM) to solve contrast enhancement problem by maximizing the expected contrast gain subject to an upper limit on tone distortion. Another line of attack is to increase contrast through edge enhancement and high-boost filtering [8, 9]. For example, in [9] the fractional filter is used to promote the variance of texture so as to enhance the image. We differ from these approaches in that a graph-based enhancement operator is used to boost contrast in the signal, where the edge weights in the graph are computed robustly.

With the recent advances in GSP, newly developed GSP tools such as *graph Fourier transforms* (GFT) [4] are now being used for traditional image processing tasks such as image compression [10, 11], denoising [12, 13, 14] and interpolation [15, 16] with demonstrable gains. The key to much of these previous work is that with appropriate edge weights, a target graph-signal contains mostly low graph frequencies and thus can be compactly represented. In contrast, in our work we construct a *dual graph* to represent edges in the original graph; edge weights in the original graph will be the target graph-signal in the dual graph. We then assume low graph frequencies to perform denoising of edge weights in the dual graph.

3. PROBLEM FORMULATION

Let $\mathbf{x}$ and $\mathbf{y}$ be the captured (sub-)image under poor light conditions and the restored (sub-)image in vector form, respectively. We perform joint contrast enhancement and denoising by minimizing the following objective function:

$$\min_{\{\mathbf{y},\boldsymbol{\alpha}\}} \|\mathbf{y}-(\mathbf{I}+\mathbf{H})\mathbf{x}\|_2^2+\lambda\left(\|\mathbf{y}-\boldsymbol{\Phi}\boldsymbol{\alpha}\|_2^2+\gamma\|\boldsymbol{\alpha}\|_1\right) \quad (1)$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{H}$ is the (high-pass) *graph Laplacian matrix* (to be discussed in details in Section 3.1). The first term in the objective function enables contrast enhancement via the Laplacian operator $\mathbf{H}$. The last two terms describe a standard sparse coding formulation to denoise the enhanced image $\mathbf{y}$ with respect to an online learned dictionary $\boldsymbol{\Phi}$. λ and γ are chosen parameters to trade off between contrast enhancement and denoising, and between sparse representation fidelity and sprasity, respectively. From (1), we observe that the two optimization variables $\mathbf{y}$ and $\boldsymbol{\alpha}$ are interdependent; reconstructed image $\mathbf{y}$ must be close to an enhanced version of observed $\mathbf{x}$ *and* to a sparse representation $\boldsymbol{\Phi}\boldsymbol{\alpha}$ at the same time.

Instead of (1), one can consider an alternative formulation:

$$\min_{\{\mathbf{y},\boldsymbol{\alpha}\}} \|\mathbf{y}-(\mathbf{I}+\mathbf{H})\boldsymbol{\Phi}\boldsymbol{\alpha}\|_2^2+\lambda\left(\|\mathbf{x}-\boldsymbol{\Phi}\boldsymbol{\alpha}\|_2^2+\gamma\|\boldsymbol{\alpha}\|_1\right) \quad (2)$$

Unlike (1), in this formulation, observed $\mathbf{x}$ is first denoised via sparse coding (second term), before the sparsified result $\boldsymbol{\Phi}\boldsymbol{\alpha}$ is contrast-enhanced (first term). In fact, the optimal $\mathbf{y}$ given $\boldsymbol{\alpha}$ is simply $(\mathbf{I}+\mathbf{H})\boldsymbol{\Phi}\boldsymbol{\alpha}$, which means the optimization in (2) is *separable*: without loss of optimality, $\boldsymbol{\alpha}$ can be first solved independently via sparse coding, before $\mathbf{y}$ is computed given $\boldsymbol{\alpha}$. This is the conventional separate denoising / contrast-enhancement approach described in the Introduction. *We argue that the separate denoising / contrast-enhancement approach is sub-optimal in general*. Image denoising is the process of optimizing the tradeoff between smoothing (to remove additive noise) and preserving details. An intermediate denoised image is thus very likely over-smoothed in some parts, eliminating important spatial details and rendering the contrast-enhancing operator $\mathbf{H}$ ineffective.

In contrast, in (1) the sparse coding driven denoising is applied to the enhanced image $\mathbf{y}$ instead. This ensures that there is no over-smoothing prior to contrast enhancement. On the other hand, it is now possible to amplify noise rather than original details of the image via operator $\mathbf{H}$, which will make the subsequent denoising task more difficult. Our solution is to properly design contrast-enhancement operator $\mathbf{H}$ so that only spatial locations with reliably detected textural details are enhanced. We discuss the design of $\mathbf{H}$ next.

3.1. Graph Laplacian Operator

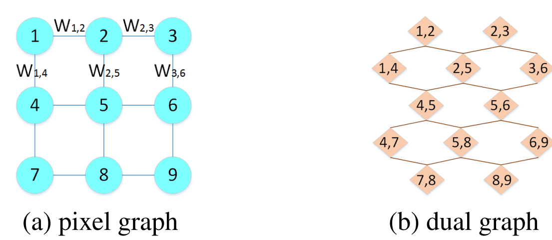


Fig. 1. Examples of graphical representation of: (a) pixels in a target patch connected by edges with weights; (b) inter-pixel similarities in a target patch. (b) is the dual graph of (a).

In GSP for image processing [11, 12, 13, 16, 17], a graph $\mathcal{G}$ is used to model inter-pixel correlation or similarities for a target pixel patch. Typically, a four-connected graph is employed to connect each pixel (represented as nodes in $\mathcal{G}$) to its immediately neighbors vertically and horizontally. Fig. 1(a) illustrates a graph representing a 3×3 pixel patch. An edge weight $w_{i,j}$ between two connected nodes v_i and v_j is conventionally computed using the pixel intensity difference, modulated by a Gaussian kernel, *i.e.*,

$$w_{i,j} = \exp\left\{-\|I_i - I_j\|_2^2/\sigma^2\right\} \quad (3)$$

where I_i is the intensity value at pixel i, and σ is a Gaussian parameter. $w_{i,j} = 0$ if v_i and v_j are not connected.

Having computed edge weights $w_{i,j}$, one can define an *adjacency matrix* $\mathbf{A}$ where $A_{i,j} = w_{i,j}$, and a diagonal *degree matrix* $\mathbf{D}$ where $D_{i,i} = \sum_j A_{i,j}$. A *combinatorial* or *unnormalized graph Laplacian* $\mathbf{L}$ is defined as the difference between $\mathbf{D}$ and $\mathbf{A}$ [4]: $\mathbf{L} = \mathbf{D} - \mathbf{A}$. A *normalized* Laplacian is simply $\mathbf{L}$ scaled by the degree matrix:

$$\mathbf{H} = \mathbf{D}^{-\frac{1}{2}}\mathbf{L}\mathbf{D}^{-\frac{1}{2}} \tag{4}$$

To avoid excessive signal amplification, we employ the normalized Laplacian $\mathbf{H}$ as the contrast enhancement operator. The crux in the design of $\mathbf{H}$ is in the computation of edge weights $w_{i,j}$ when the observed pixel values are corrupted by noise and hence not reliable. We discuss this next.

3.2. Edge Weight Filtering using Dual Graph

To compute a set of reliable weights $w_{i,j}$ for the contrast enhancement operator $\mathbf{H}$, we perform the following procedure. Observing that computing the observed pixel difference $I_i - I_j$ directly will result in twice the noise variance of a single noise-corrupted pixel, as a pre-processing step, we first perform *bilateral filter* (BF) [2] on observed $\mathbf{x}$, resulting in a locally denoised version $\mathbf{s}$. Using $\mathbf{s}$, we compute weights $w_{i,j}$ using (3). We then perform graph-based low-pass to remove *noise in the computed weights* $w_{i,j}$. The idea is that edge weights themselves are correlated locally. For example, if a pixel patch contains portions of both foreground and background regions, then edges connecting pixels exclusively in the foreground / background would contain large weights (since pixels in foreground / background are similar), while edges that connect foreground pixels in background pixels would contain small weights. The edge weights themselves thus result in a piecewise smooth signal, and performing graph-based low-pass filtering can remove noise among the weights as done in [12].

Specifically, we first construct a *dual graph*, where each node now represents an edge with weight $w_{i,j}$ in the original graph. See Fig. 1(b) for the dual graph to the original graph in Fig. 1(a). Each node is connected to its neighbors representing original edges that are diagonal from the represented edge. We then compute *link* weights in the dual graph using also a Gaussian kernel reflecting the similarities of the represented edge weights. Computed link weights lead to another combinatorial Laplacian $\mathcal{L}$. We then solve the following for a filtered version of the edge weights $\mathbf{z}$:

$$\min_{\mathbf{z}} \|\mathbf{w} - \mathbf{z}\|_2^2 + \lambda \mathbf{z}^T \mathcal{L} \mathbf{z} \tag{5}$$

The filtered edge weights $\mathbf{z}$ are then used to compute the contrast enhancing graph Laplacian $\mathbf{H}$ described previously.

4. OPTIMIZATION

After obtaining the Laplacian operator $\mathbf{H}$, we now describe how to compute the optimal solution to optimization in Eq. (1). The objective function is not jointly convex in $\mathbf{y}$ and $\boldsymbol{\alpha}$, but is convex in one variable if the other is fixed. Therefore, we can employ an alternating procedure to optimize these variables iteratively. Specifically, to tackle the objective function we separate the objective function into two sub-problems, which we describe in details next. This procedure is repeated until convergence or after a maximum number of iterations T has been reached. In what follows, we will describe the initialization process, three sub-problems and their optimal solutions.

4.1. Initialization Process

We first perform *bilateral filter* on the input image $\mathbf{x}$ to get an initially denoised version $\widehat{\mathbf{x}}$. Then, the initial Laplacian operator $\mathbf{H}_0$ is computed using the dual graph strategy described in the previous section. Given $\widehat{\mathbf{x}}$ and $\mathbf{H}$, we can compute an initial estimate of $\mathbf{y}$: $\mathbf{y}_0 = (\mathbf{I} + \mathbf{H}_0)\mathbf{x}$.

4.2. Optimization with respect to $\boldsymbol{\alpha}$

Given $\mathbf{y}$, the optimal $\boldsymbol{\alpha}$ can be derived by solving the following problem:

$$\arg\min_{\boldsymbol{\alpha}} \left\{ \|\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\alpha}\|_2^2 + \gamma \|\boldsymbol{\alpha}\|_1 \right\}. \tag{6}$$

This is a well-known standard sparse coding problem. The optimization solution of $\boldsymbol{\alpha}$ can be effectively and efficiently solved using a fast ℓ_1-minimization algorithm, known as *Augmented Largrangian Methods* (ALM) [18].

Dictionary plays a critical role in the above sparse coding problem. In our method, locally adaptive dictionaries are learned because natural images typically exhibit non-stationary statistics, consisting of many heterogeneous regions of significantly different geometric structures or statistical characteristics. Specifically, for a local patch $\mathbf{y}_i$, we take advantage of the non-local self-similarity of natural images, and collect similar patches by non-local patch grouping (NLPG) in the training data set. The NLPG procedure guarantees that only the similar sample blocks are used in dictionary learning. The resulted similar patch group is then subject to principle component analysis (PCA). PCA generates the dictionary $\boldsymbol{\Phi}_i$ whose atoms are the eigenvectors of the covariance matrix of Ψ_i. Finally, all sub-dictionaries are concatenated together to form an over-complete dictionary.

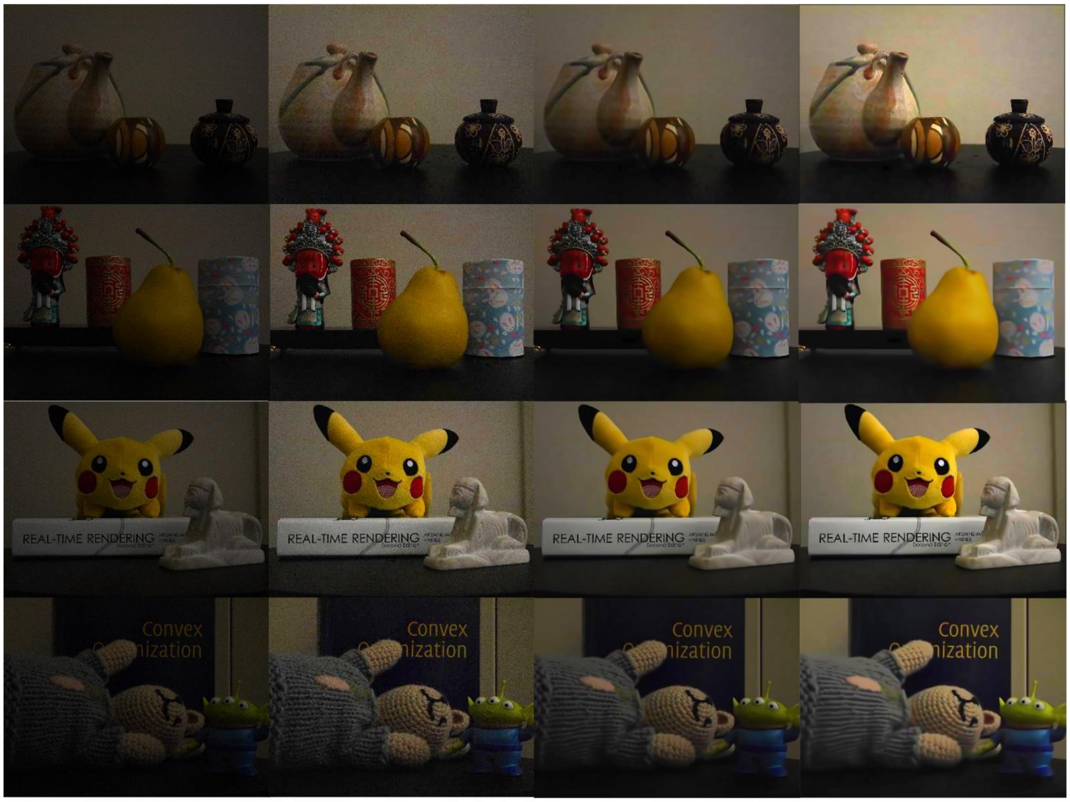
4.3. Optimization with respect to $\mathbf{y}$

Given $\boldsymbol{\alpha}$, the optimization problem with respect to $\mathbf{y}$ reduces to the following form:

$$\min_{\mathbf{y}} \|\mathbf{y} - (\mathbf{I} + \mathbf{H})\mathbf{x}\|_2^2 + \lambda \|\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\alpha}\|_2^2. \tag{7}$$

Setting the first-order derivative to zero:

$$\frac{\partial J}{\partial \mathbf{y}} = 2(\mathbf{y} - (\mathbf{I} + \mathbf{H})\mathbf{x}) + 2\lambda(\mathbf{y} - \boldsymbol{\Phi}\boldsymbol{\alpha}) = 0, \tag{8}$$



(a) (b) (c) (d)

Fig. 2. Results of different methods on four images. (a) Degraded image. (b) OCTM. (c) OCTM+BM3D, separately. (d) Our joint method.

the optimal solution can be finally computed as:

$$\mathbf{y}^* = \frac{(\mathbf{I} + \mathbf{H})\,\mathbf{x} + \lambda \boldsymbol{\Phi} \boldsymbol{\alpha}}{1 + \lambda}. \tag{9}$$

5. EXPERIMENTATION

We present experimental results in this section. We compare our proposed joint denoising / contrast-enhancement method with contrast enhancement only, and separate contrast enhancement / denoising methods. We choose OCTM [3] as the competing contrast enhancement algorithm, and BM3D [19] as the competing denoising algorithm. Both of them can be considered state-of-the-art. Our method involves two parameters λ and γ, which are set as 0.3 and 0.001 in practical implementation, respectively.

Fig. 2 shows the original images captured under poor lighting in (a) and processed images using different methods in (b) to (d). We see that the captured images in (a) are dark and corrupted by noise. As shown in (b), OCTM can improve the contrast, but it also enhances the noise. Images in (c) show the results of performing OCTM followed by BM3D separately. We can observe that BM3D suppresses noise effectively. However, it also removes enhanced details by OCTM, therefore reduces the effect of OCTM. We see that the resulting images in (c) is darker compared with that in (b). We observe that our proposed method achieves the best overall visual effect: it not only enhances the contrast, but also suppresses noises. Our method casts denoising and enhancement into a unified framework, and therefore can achieve a good tradeoff between high-frequency enhancement and noise removal for the final images. Our method works better than the separate approach of employing state-of-the-art OCTM and BM3D in two stages.

6. CONCLUSION

Images captured in poor lighting conditions require denoising and contrast enhancement post-processing. In this paper, we propose to perform both tasks simultaneously via a joint denoising and contrast enhancement framework. The crux of the optimization rests on the design of the enhancement operator, so that only textural details of the original image are enhanced and not the acquisition noise. We propose a graph-based enhancement operator, where the graph weights are computed robustly via a graph-signal smoothness prior. Experimental results show cleaner and better contrast-enhanced images compared to previous proposals.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

[1] A. Kirmani et al., "First-photon imaging," in *Science (online)*, November 2013, vol. 343, no.6166, pp. 58–61.

[2] C. Tomasi and R. Manduchi, "Bilateral filtering for gray and color images," in *IEEE Int'l Conf. Computer Vision*, 1998, pp. 839–846.

[3] Xiaolin Wu, "A linear programming approach for optimal contrast-tone mapping," *Image Processing, IEEE Transactions on*, vol. 20, no. 5, pp. 1262–1272, May 2011.

[4] D.I. Shuman, S.K. Narang, P. Frossard, A. Ortega, and P. Vandergheynst, "The emerging field of signal processing on graphs: extending high-dimensional data analysis to networks and other irregular domains," *IEEE Signal Processing Magazine*, vol. 30, no. 3, pp. 83–98, 2013.

[5] A. Buades, B. Coll, and J-M Morel, "A non-local algorithm for image denoising," in *IEEE Int'l Conf. Computer Vision and Pattern Recognition*, 2005, vol. 2, pp. 60–65.

[6] M. Elad and M. Aharon, "Image denoising via sparse and redundant representation over learned dictionaries," in *IEEE Transactions on Image Processing*, December 2006, vol. 15, no.12.

[7] T. Arici, S. Dikbas, and Y. Altunbasak, "A histogram modification framework and its application for image contrast enhancement," *Image Processing, IEEE Transactions on*, vol. 18, no. 9, pp. 1921–1935, Sept 2009.

[8] Zeev Farbman, Raanan Fattal, Dani Lischinski, and Richard Szeliski, "Edge-preserving decompositions for multi-scale tone and detail manipulation," in *ACM SIGGRAPH 2008 Papers*, New York, NY, USA, 2008, SIGGRAPH '08, pp. 67:1–67:10, ACM.

[9] Yi-Fei Pu, Ji-Liu Zhou, and Xiao Yuan, "Fractional differential mask: A fractional differential-based approach for multiscale texture enhancement," *Image Processing, IEEE Transactions on*, vol. 19, no. 2, pp. 491–511, Feb 2010.

[10] G. Shen, W.-S. Kim, S.K. Narang, A. Ortega, J. Lee, and H. Wey, "Edge-adaptive transforms for efficient depth map coding," in *IEEE Picture Coding Symposium*, Nagoya, Japan, December 2010.

[11] W. Hu, G. Cheung, X. Li, and O. Au, "Depth map compression using multi-resolution graph-based transform for depth-image-based rendering," in *IEEE International Conference on Image Processing*, Orlando, FL, September 2012.

[12] W. Hu, X. Li, G. Cheung, and O.C. Au, "Depth map denoising using graph-based transform and group sparsity," in *IEEE Int'l Workshop on Multimedia Siging Processing*, 2013.

[13] A. Kheradmand and P. Milanfar, "A general framework for kernel similarity-based image denoising," in *IEEE Global Conf. Sig. Info. Process.*, 2013, pp. 415–418.

[14] J. Pang, G. Cheung, W. Hu, and O.C. Au, "Redefining self-similarity in natural images for denoising using graph signal gradient," in *APSIPA ASC*, 2014.

[15] Y. Mao, G. Cheung, A. Ortega, and Y. Ji, "Expansion hole filling in depth-image-based rendering using graph-based interpolation," in *IEEE International Conference on Acouistics, Speech and Signal Processing*, Vancouver, Canada, May 2013.

[16] Y. Mao, G. Cheung, and Y. Ji, "Graph-based interpolation for dibr-synthesized images with nonlocal means," in *Symposium on Graph Signal Processing in IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, Austin, TX, December 2013.

[17] X. Liu, D. Zhai, D. Zhao, G. Zhai, and W. Gao, "Progressive image denoising through hybrid graph laplacian regularization: a unified framework," *IEEE Trans. Imag. Process.*, vol. 23, no. 4, pp. 1491–1503, 2014.

[18] AY. Yang, Zihan Zhou, AG. Balasubramanian, S.S. Sastry, and Yi Ma, "Fast ℓ_1-minimization algorithms for robust face recognition," *Image Processing, IEEE Transactions on*, vol. 22, no. 8, pp. 3234–3246, Aug 2013.

[19] K. Dabov, A Foi, V. Katkovnik, and K. Egiazarian, "Image denoising by sparse 3-d transform-domain collaborative filtering," *Image Processing, IEEE Transactions on*, vol. 16, no. 8, pp. 2080–2095, Aug 2007.