Homework Assignment #2
Due: September 22, 2025 at 5:00 p.m.

1. A no-writing (single-tape) Turing machine is one that never changes any character written on its tape. (Thus, in the transition function of such a machine, if $\delta(q, a) = (q', a', d)$ then $a = a'$.) Let L be a language. Assume there is a no-writing Turing machine M that decides whether its input string is in L.

[3] **(a)** Prove that there exists a constant k such that, for every input string x, M never visits the same square of the tape more than k times.
Hint: Think about the sequence of states M is in when it visits that square.

[4] **(b)** Show that you can construct a no-writing Turing machine M' that decides L without ever moving beyond the first $n + 2$ squares of the tape on any input of length n.

[2] **(c)** Prove that $L \in \mathit{TIME}(n)$.

[5] **2.** Recall that if L_1 and L_2 are languages, then $L_1 \cdot L_2 = \{x_1x_2 : x_1 \in L_1 \text{ and } x_2 \in L_2\}$.
Prove that if L_1 and L_2 are in P then $L_1 \cdot L_2 \in P$.