Homework Assignment #1
Due: September 15, 2025 at 5:00 p.m.

Before working on this assignment, you should read the course policy on academic honesty on the course web page at www.eecs.yorku.ca/course/4115.

1. Sulev claims that for every pair of functions f, g mapping natural numbers to natural numbers, either f is $O(g)$ or g is $O(f)$. Is Sulev's claim true or false? Prove your answer is correct.

2. We discussed the RAM model in class. There are some notes about it under readings for Sep 8 on the course web page. For this question, we use the unit cost model, where each RAM instruction can be done in one unit of time.

We saw that a RAM programme that takes $T(n)$ steps on inputs of size n can be simulated on a multitape Turing machine in $O((T(n))^2(T(n)+n))$ time.

(a) Now, suppose we add multiplication to the instruction set of the RAM model, so that an instruction can set $R[i] \leftarrow R[j] \cdot R[k]$. What changes in the simulation argument? (Is the simulation still possible? If so, does the bound on the number of steps that the simulation takes still hold?) Explain why your answer is correct.

(b) Many chips do provide instructions that can multiply the values stored in two memory locations in $O(1)$ clock cycles. Does your answer to (a) say anything about how good Turing machines are as a model for measuring the a problem's time complexity on a real computer? Explain.