Test 2

First Name: _____

Last Name: _____

Student Number: _____

*This test lasts **75 minutes**. No aids allowed.*
Make sure your test has 6 pages, including this cover page.
*Answer in the space provided. (If you need more space, use page 6 and indicate **clearly** which part of your work should be marked.)*
Write legibly.

Question 1	/2
Question 2	/5
Question 3	/4
Question 4	/5
Question 5	/4
Total	/20

[2] **1.** Let L be a language. Explain the difference between a Turing machine that decides L and a Turing machine that recognizes L.

[5] **2.** Let L_1 and L_2 be languages over the alphabet Σ.

(a) Give a precise definition of what it means to say $L_1 \leq_m L_2$.

(b) Show that if L is recognizable and $L \leq_m \overline{L}$, then L is decidable.

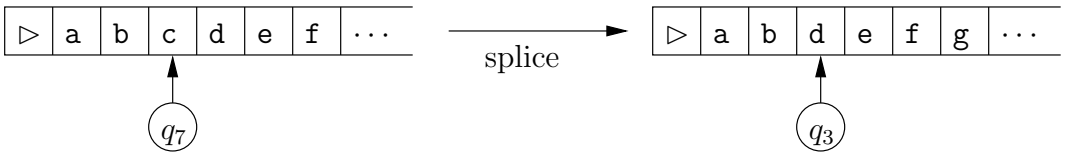
[4] **3.** Let $L_3 = \{\langle M, w_1, w_2\rangle : M$ is a Turing machine that accepts string w_1 or w_2 (or both)$\}$. Give a deterministic algorithm that recognizes L_3. Explain why your answer is correct.

[5] **4.** Let $L_4 = \{\langle M, k\rangle : k \in \mathbb{N} \text{ and } M \text{ is a Turing machine that accepts no string of length } k\}$.

(a) Is $\overline{L_4}$ recognizable? You do not have to prove your answer is correct.

(b) Prove that L_4 is undecidable.

[4] **5.** A splicing Turing machine (STM) is similar to an ordinary Turing machine. The only difference is that it has the additional ability to perform a splice, which cuts the square at the location of the STM's head out of the tape and then glues the two remaining pieces of the tape together. The head of the STM then moves to the square either to the left or right of the removed square, depending on the transition specified. An example of a splice step is shown below, if the transition function of the STM specifies that $\delta(q_7, \texttt{c}) = (q_3, \text{splice}, R)$.



Let M_s be any STM. Explain (at a high level) how you could construct an ordinary Turing machine M that simulates the actions of M_s. Your explanation should be in clear, precise English.

This (nearly) blank page is for additional workspace, if you need it.