Quiz 4

First Name: _____

Last Name: _____

Student Number: _____

*This test lasts **60 minutes**. No aids allowed.*
Make sure your test has 4 pages, including this cover page.
*Answer in the space provided. (If you need more space, use the reverse side of the page and indicate **clearly** which part of your work should be marked.)*
Write legibly.

Question 1	/5
Question 2	/6
Question 3	/4
Question 4	/4
Total	/19

[5] **1.** In class, we studied the following context-free grammar G. G has just one variable S. The set of terminals is $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, and the grammar has the following rules.

$$\begin{aligned} S &\to \mathtt{a}S\mathtt{b}S \\ S &\to \mathtt{b}S\mathtt{a}S \\ S &\to \varepsilon \end{aligned}$$

(a) Give a derivation of the string `aabb` using the grammar G.

(b) Show that the grammar G is ambiguous.

[6] **2.** Let G_2 be the context-free grammar having a single variable S, terminal set $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and the following three rules.

$$S \to \mathtt{a}S\mathtt{b} \mid \mathtt{a}S\mathtt{bb} \mid \varepsilon$$

Fill in the missing pieces below to obtain a **careful** proof that every string in the language $L_2 = \{\mathtt{a}^i\mathtt{b}^j : i, j \in \mathbb{N} \text{ and } i \leq j \leq 2i\}$ can be generated from S in the grammar G_2.

Proof by induction on n, the number of a's in the string.
Base case ($n = 0$): Let x be any string in L_2 that contains no a's.
Prove that x can be generated from S.

Induction step Let $n \geq 1$. Assume that all strings in L_2 that contain _____ a's can be generated from S. Let x be any string in L_2 that contains exactly n a's. Then, $x = \mathtt{a}^n\mathtt{b}^j$, for some j. Prove that x can be generated from S by considering two cases.
Case 1 ($n = j$):

Case 2 ($n \neq j$):

[4] **3.** Let $L_3 = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k : i, j, k \in \mathbb{N} \text{ and } i + j = k\}$. Give a context-free grammar for L_3. You do *not* have to prove your answer is correct, but you should give, for each variable, a precise description of the strings generated by that variable.

[4] **4.** Briefly describe, in English, how a pushdown automaton could be built to decide the language $L_4 = \{\mathtt{a}^i\mathtt{b}^{2i} : i \geq 0\}$.