Quiz 2

First Name: _____

Last Name: _____

Student Number: _____

*This test lasts **60 minutes**. No aids allowed.*
Make sure your test has 4 pages, including this cover page.
*Answer in the space provided. (If you need more space, use the reverse side of the page and indicate **clearly** which part of your work should be marked.)*
Write legibly.

Question 1	/5
Question 2	/3
Question 3	/5
Question 4	/4
Question 5	/3
Total	/20

[5] **1.** Let $L_1 = \{w \in \{\mathtt{a}, \mathtt{b}\}^* : \text{the number of } \mathtt{b}\text{'s in } w \text{ is not equal to } 2\}$. Draw the transition diagram of a *deterministic* finite automaton for L_1. Then, for each state q of your automaton, describe (in precise English) which strings take the machine from the initial state to q.

[3] **2.** Give a high-level description (in plain but precise English) of how to prove that the regular languages are closed under concatenation. (I.e., how to prove that if languages L_1 and L_2 are regular, then $L_1 \cdot L_2$ must also be regular.) Your answer must fit inside the box below. Anything written outside the box will be ignored.

[5] **3.** In class, we saw that if L_1 and L_2 are regular, then so is $L_1 \cup L_2$. Use this fact to prove that, for all $n \geq 1$, if $L_1, L_2, \ldots, L_n$ are all regular languages, then the language $L_1 \cup L_2 \cup \cdots \cup L_n$ is also regular.
Hint: use mathematical induction.

[4] **4.** Let $\Sigma = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$. A string in Σ^* represents two natural numbers in binary representation. Let L_4 be the language of all strings where the number represented by the top row equals one plus the number represented by the bottom row. For example, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \in L_4$ because the bottom row (`100111`) represents 39 in binary and the top row (`101000`) represents 40, and $40 = 39 + 1$. Draw the transition diagram of a deterministic finite automaton for L_4. You do not have to prove your answer is correct.

[3] **5.** Let $N = (Q, \Sigma, q_0, \delta, F)$ be a non-deterministic finite automaton.
Let $N' = (Q, \Sigma, q_0, \delta, Q - F)$.
Let L and L' be the languages accepted by N and N', respectively.
Is it true that, for every NFA N, $L' = \overline{L}$?
Answer clearly: YES or NO. Briefly explain why your answer is correct.