Homework Assignment #1
Due: Monday, April 6 2026 at noon

Note the unusual deadline; I would like to hand out solutions on April 6, so you must submit your solution before noon that day.

[6] **1.** Consider the following grammar with starting symbol S.

$$S \to \texttt{0}S\texttt{11} \mid S\texttt{1} \mid \texttt{0}$$

Let $L = \{\texttt{0}^i\texttt{1}^j : i \geq 1 \text{ and } j \geq 2i - 2\}$.

Give a formal proof of the following claim: For all $n \geq 0$, every string of length n in L can be generated by the grammar.

2. Let $B(n)$ be the binary representation of the positive integer n with no leading 0's. For example, $B(23) = \texttt{10111}$. Recall that x^R is the reverse of a string x. So $(B(23))^R = \texttt{11101}$. Let $L = \{B(n)\texttt{\#}(B(2n+3))^R : n \geq 1\}$. For example, $\texttt{10111\#100011} \in L$, since $B(23) = \texttt{10111}$ and $B(2 \cdot 23 + 3) = B(49) = \texttt{110001}$ and $(B(49))^R = \texttt{100011}$, so $\texttt{10111\#100011} = B(23)\texttt{\#}(B(2 \cdot 23 + 3))^R$.

[2] **(a)** How do you have to modify the string $B(n)$ to get the string $B(2n+3)$? Your description of the modification should work for all positive integers n.

[4] **(b)** Draw the transition diagram of a PDA for L. Explain, in English, what your PDA does. (This explanation will be used to help the TA understand your PDA, and give you partial credit if your PDA is incorrect, but you had some of the right ideas. So, be sure to write your explanation clearly.)