Homework Assignment #6
Due: October 31, 2025 at 11:59 p.m.

1. Consider a collection of synchronous processors that are arranged in a ring network. Each processor receives an input bit (0 or 1). The goal is to have every processor output the xor of all the input bits.

[4] **(a)** Suppose processors are anonymous (no ids; all processors run identical code) and know that the size of the ring is n. Give a deterministic algorithm to compute the xor of their input bits. How many messages does your algorithm use in the worst case? (The fewer, the better.)

[4] **(b)** Give a simple deterministic algorithm to solve the xor problem if all processors have unique ids, but do not know the size of the ring. How many messages does your algorithm use as a function of the ring size?

Hint: you can use any algorithm described in class as a subroutine.

2. Consider a synchronous ring of n anonymous processors, where n is unknown. We design a randomized leader election algorithm as follows. Suppose each processor chooses a random id from a large enough range that the probability that two processors choose the same id is negligible. Each processor sends its id to its clockwise neighbour. Whenever a processor receives a message, it forwards it to its clockwise neighbour if and only if it is less than any id it has seen before. If a processor gets a message containing its own id, it outputs "leader". If a processor gets a message containing a smaller id than its own, it outputs "follower".

[3] **(a)** Argue that the expected distance the ith smallest id will travel is n/i.

[2] **(b)** Give a good upper bound on the expected number of messages used by the algorithm.

[2] **(c)** What happens to the algorithm if two processors do choose the same id?