Homework Assignment #8
Due: December 1, 2025 at 5:00 p.m.

1. A graph is *planar* if it is possible to draw it on (one side of) a flat piece of paper without any pair of edges crossing each other. For example, a clique of 4 nodes is planar, but a clique of 5 nodes is not.

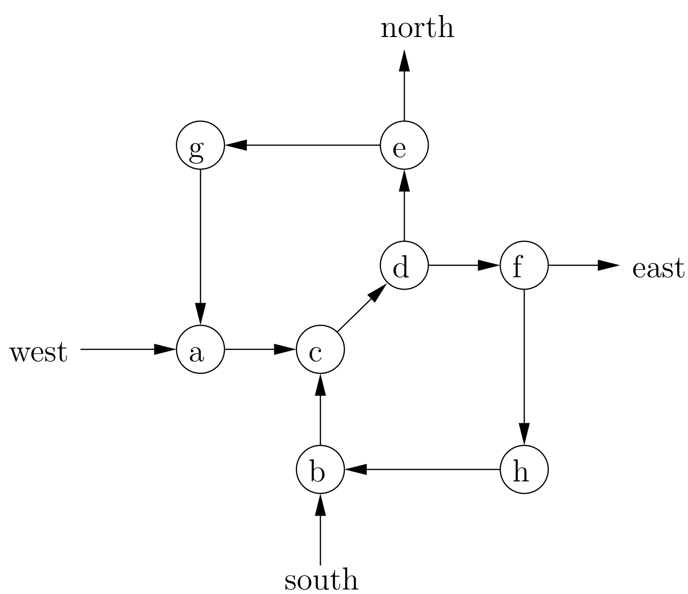
PLANAR-GEOGRAPHY is the same problem as GEOGRAPHY, with the additional restriction that the graph that that the game is played on is planar.

The graph constructed by the reduction from TQBF to GEOGRAPHY is not planar (at least for some formulas). So, that reduction does not directly prove that TQBF is reducible to PLANAR-GEOGRAPHY. The goal of this proof is to modify the proof that GEOGRAPHY is PSPACE-complete in order to show that PLANAR-GEOGRAPHY is also PSPACE-complete.

[1] **(a)** Explain why PLANAR-GEOGRAPHY is in PSPACE.

[3] **(b)** The graph used in the reduction TQBF $\leq_P$ GEOGRAPHY is illustrated in Figure 8.16 of the textbook. Describe a different way of drawing this graph on a sheet of paper such that, for every pair of edges that cross each other, the two edges will not *both* be used when the game is played. Your construction should work for any formula that is an input for TQBF (not just the one shown in Figure 8.16). Explain why your answer is correct.

[3] **(c)** Consider the following planar graph fragment.



Suppose a player A starts by using the edge labelled west. Show that both players *must* play so that player A uses the edge labelled east. Similarly, if a player A starts by using the edge labelled south, then player A must also use the edge labelled north.

[1] **(d)** What happens if a player starts by using the edge labelled west in the graph in part (c) and then later a player uses the edge labelled south?

[4] **(e)** Show that PLANAR-GEOGRAPHY is PSPACE-complete. (Your answer should use parts (a), (b) and (c).)