York University EECS 4115A/5115A Eric Ruppert November 10, 2025

Homework Assignment #7
Due: November 17, 2025 at 5:00 p.m.

[2] **1.** In class, we showed that REACHABILITY is NL-complete.
Explain why UNREACHABILITY is also NL-complete.

2. Consider the following 2INEQUALITIES problem. Given a set of inequalities involving n variables $x_1, \ldots, x_n$, each of the form $x_i \leq x_j$ or $x_i \leq v$ or $x_i \geq v$, where $v \in \mathbb{N}$, determine whether there are values of $x_1, \ldots, x_n$ that satisfy all of the inequalities.

[2] **(a)** What is the answer to 2INEQUALITIES for the following input? Explain why your answer is correct.

$$\begin{array}{rcl} x_1 & \leq & x_2 \\ x_3 & \leq & x_4 \\ x_6 & \leq & x_1 \\ x_5 & \leq & x_3 \\ x_4 & \leq & 4 \\ x_1 & \leq & 17 \\ x_5 & \geq & 12 \end{array}$$

[4] **(b)** Claim: the answer for a given instance is no if and only if there is a sequence of inequalities $v_1 \leq x_{i_1} \leq x_{i_2} \leq \cdots \leq x_{i_k} \leq v_2$ where v_1 and v_2 are natural numbers with $v_1 > v_2$.

(i) Prove the "if" direction of the claim.

(ii) Prove the "only if" direction of the claim by proving the contrapositive. Assume there is no such sequence of inequalities, design a solution by setting the value of each x_i as follows.

- if there is no sequence of inequalities of the form $x_i \leq x_{i_1} \leq x_{i_2} \leq \ldots x_{i_k} \leq v$, then set x_i to the largest natural number that appears in any of the inequalities;
- otherwise, set x_i to the minimum value v such that there is a sequence of inequalities $x_i \leq x_{i_1} \leq x_{i_2} \leq \ldots x_{i_k} \leq v$.

Show that this solution satisfies every inequality.

[3] **(c)** Show that 2INEQUALITIES is in NL.
Hint: use the trick from Assignment 6 so that you do not actually have to write down a constant that appears in one of the inequalities on the work tape.

[4] **(d)** Show that 2INEQUALITIES is NL-complete.
Hint: Use the result of Question 1.