

Homework Assignment #5
Due: November 3, 2025 at 5:00 p.m.

- [4] 1. Recall that if L_1 and L_2 are languages, then $L_1 \cdot L_2 = \{x_1x_2 : x_1 \in L_1 \text{ and } x_2 \in L_2\}$. Prove that if L_1 and L_2 are in coNP then L_1L_2 is in coNP.
- [9] 2. Consider the following problem of partitioning a set into two groups so that the total of the squared sums of the two groups is small.
Given natural numbers $x_1, \dots, x_n, k$, do there exist disjoint sets A and B such that $A \cup B = \{1, 2, \dots, n\}$ and $(\sum_{i \in A} x_i)^2 + (\sum_{i \in B} x_i)^2 \leq k$?
This problem is known to be NP-complete.
Use dynamic programming to give a pseudopolynomial time algorithm that solves the problem and explain why it works.
Your solution should include the following elements.
- The size of the array used to store solutions to subproblems.
 - A description, in precise English, of the subproblem whose solution is stored in each location of the array.
 - A recurrence that you can use to fill in the array, including any base case(s).
 - What order you would fill in the entries of the array.
 - How you would use information from the array to output the answer to the original problem.
 - Running time of your algorithm.