Homework Assignment #6
Due: March 7, 2025 at 5:00 p.m.

1. We wish to keep track of a sequence $x_1, x_2, \ldots, x_n$ of integers. We are interested in finding the maximum sum of any group of consecutive elements in the sequence. For example, if the sequence is $3, -12, 2, 5, -4, 6, -8, 2, 1$, then the maximum possible sum is $2 + 5 - 4 + 6 = 9$. The group of consecutive elements can be empty, in which case the sum is 0. For example, if the sequence is $-3, -6, -7$ then the maximum possible sum of a group of consecutive elements is 0.

The goal of this question is to use an augmented red-black tree to implement the following operations efficiently.

- INSERT(x, i) inserts the integer x after the ith element of the sequence. This operation changes the sequence from $x_1, x_2, \ldots, x_n$ to $x_1, x_2, \ldots, x_i, x, x_{i+1}, x_{i+2}, \ldots, x_n$. If $i = 0$, this operation inserts x before the first element of the sequence.
- DELETE(i) deletes the ith element of the sequence. This operation changes the sequence from $x_1, x_2, \ldots, x_n$ to $x_1, x_2, \ldots, x_{i-1}, x_{i+1}, x_{i+2}, \ldots, x_n$.
- MAXSUM returns the maximum sum of any group of consecutive elements in the sequence.

[2] **(a)** What are the items that you store in the red-black tree? How are the items ordered in the tree?

[5] **(b)** Define the additional fields you will add to each of the nodes of the RBT, and what the fields are supposed to represent.

Hint: First, think about what augmentation is needed to implement INSERT and DELETE. Then, think about what augmentation is needed so that MAXSUM operations can be done in $O(1)$ time. For the latter, it might be helpful to consider the problem of finding the maximum sum of any group of consecutive elements that includes x_i. What pieces of information about $x_1, \ldots, x_{i-1}$ and about $x_{i+1}, \ldots, x_n$ would be useful to find this quantity?

[5] **(c)** Describe how you can compute the new fields of a node v using only information stored in v and its children in constant time.

[5] **(d)** Describe how to implement the three operations using your augmented RBT. Try to make use of RBT operations that were described in class or in the textbook. (If you use those, you do not have to explain how it works.) You can also use Theorem 17.1 of the textbook.

[3] **(e)** Give good upper bounds on the worst-case time needed to perform each of the three operations, and on the space used by your data structure. Use big-O notation to state your bounds in terms of n, the length of the sequence. Briefly explain why your answers are correct.