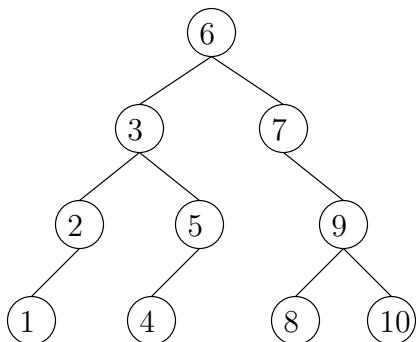


**Homework Assignment #5**  
**Due: February 28, 2025 at 5:00 p.m.**

- [3] 1. Suppose we shuffle the values  $1, 2, 3, \dots, 10$  into a random order and insert them into an initially-empty BST one by one. All 3,628,800 possible orders are equally likely. How many of those orderings yield the BST shown below? Explain why your answer is correct.



2. Consider the experiment of shuffling  $n$  distinct keys into a random order (with all orders equally likely). Let  $C_n$  be the total number of comparisons between keys performed when the  $n$  keys are inserted one-by-one into an initially empty BST.

- [2] (a) Let  $Z_{n,i} = \begin{cases} 1 & \text{if the root of the } n\text{-node BST contains the } i\text{th smallest key} \\ 0 & \text{otherwise} \end{cases}$ .

Explain why  $C_n = \sum_{i=1}^n Z_{n,i}(C_{i-1} + C_{n-i} + n - 1)$ .

- [2] (b) Show that  $E[C_n] = n - 1 + \frac{2}{n} \sum_{i=1}^{n-1} E[C_i]$ .

- [2] (c) Show that  $\sum_{i=1}^{n-1} i \log i \leq \frac{1}{2}n^2 \log n - \frac{1}{4}n^2 + \frac{1}{4}$ .

(One technique to do this is described on page 1150 in the textbook, but you can use any technique you like.)

- [3] (d) Show that  $E[C_n]$  is  $O(n \log n)$ .