Homework Assignment #7
Due: March 14, 2025 at 5:00 p.m.

The same rules apply as for Assignment 1. (In particular, you can work in pairs, where each pair submits just one paper.)

1. Mrs. Vilve Yachke owns a fast-food restaurant that serves cabbage rolls and coffee to the citizens of Bugreslava, the capital (and only) city of Leutonia. She is looking to expand her chain into the Leutonian countryside because she knows Leutonian farmers love nothing more than going out to eat cabbage rolls and coffee. She obtains a map of the trans-Leutonian highway, which stretches 1000 km from Bugreslava, at the southern end of Leutonia, to the northern border of the country. Along the way, it passes through most of the major villages of Leutonia. There are n villages along the highway. Let d_i be the distance from Bugreslava to the ith village in kilometres. Assume the villages are sorted so that $d_1 < d_2 < \cdots < d_n$. Assume that Mrs. Yachke's original, flagship restaurant is located right at the southern terminus of the trans-Leutonian highway.

Mrs. Yachke wishes to build a chain of restaurants along the highway so that each of the n villages along the highway is within 30 km of one of her restaurants. However, she wishes to build the smallest possible number of restaurants. The restaurants do not have to be located in villages; they can be located between two villages.

[4] **(a)** Give a simple, efficient, greedy algorithm that outputs an optimal list of locations for Mrs. Yachke's restaurants. Specify the algorithm in detail.

[1] **(b)** What is the worst-case running time of your algorithm in part (a)? State your answer in terms of n using Θ notation. You may assume that distances along the highway can be represented as floating point numbers in a single word of memory.

[4] **(c)** Prove that your algorithm produces the optimal solution. In particular, say precisely what it means for an optimal solution to extend a partial solution, and prove that, after each decision made by your algorithm, the partial solution computed so far can be extended to an optimal solution.