York University EECS 3101Z Eric Ruppert January 17, 2025

# Homework Assignment #2
## Due: January 24, 2025 at 5:00 p.m.

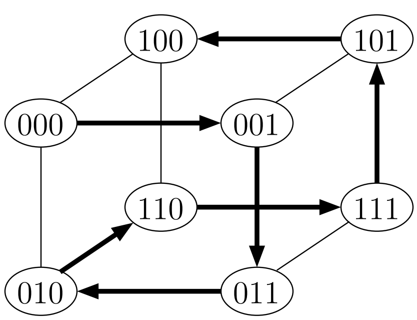
The same rules apply as for Assignment 1. (In particular, you can work in pairs, where each pair submits just one paper.)

**1.** Let $n \geq 1$. Suppose we have a collection of $N = 2^n$ computers. They are connected into a hypercube network as follows. Each computer is labelled by a binary string of length $n$. No two computers have the same label. There is a wire connecting two computers if and only if the two labels of the computers differ in exactly one bit. For example, if $n = 8$ there would be a direct connection between the computers labelled 01001001 and 01011001 because the two labels differ only in the value of their fourth bits. When $n = 3$, the network forms a cube as shown in the diagram below.

Suppose we want the computer with label $000\ldots0$ to send some information to all other computers, but we only want to transmit messages across $N-1$ of the edges. Willemina figured out how to do this and wrote a recursive algorithm to print out the order in which the information should be passed around the network.

$\textsc{List}(n)$ : list of strings
  **if** $n = 1$ **then return** $\langle 0, 1 \rangle$
  **else**
    $L \leftarrow \textsc{List}(n-1)$
    let $x_1, x_2, \ldots, x_k$ be the elements of $L$ (in order)
    **return** $\langle 0x_1, 0x_2, \ldots, 0x_{k-1}, 0x_k, 1x_k, 1x_{k-1}, \ldots, 1x_2, 1x_1 \rangle$

For example, $\textsc{List}(3)$ outputs $\langle 000, 001, 011, 010, 110, 111, 101, 100 \rangle$. Thus, according to Willemina's scheme, the message would be sent around the network as shown by the arrows below.




Prove that for all $n \geq 1$, the output of $\textsc{List}(n)$ is correct. In other words, prove that it has the following two properties.

- Every binary string of length $n$ is printed exactly once.
- If two labels appear next to each other in the list, then there is a wire between the two computers with those labels.