## Digital Logic

Logic And Verilog

## Gates



## The most obvious gates are AND and OR

We can combine them to implement any logic function

## Conventions

Zero volts is logic-0
5 volts is logic-1 (that's old fashioned TTL logic)
Unless we use negative logic
Most computers use smaller voltages now
1.5 volt is used by DDR3 memories

In this case 1.5 volt is logic-1
Due to electrical noise the logic levels are defined by a range.

## Other gates



The little circle means
"not"
NOR gate (not-OR)

NAND gate

WHAT???

WHAT???

## Truth Tables



| INP | OUT | It is the opposite of an <br> AND gate |
| :--- | :--- | :--- | :--- |
| 000 | 1 | It is a NAND gate |
| 001 | 1 |  |
| 010 | 1 |  |
| 011 | 1 |  |
| 100 | 1 |  |
| 101 | 1 |  |
| 110 | 1 |  |
| 111 | 0 |  |

## Example



## Try to figure out what this does

It is a one bit adder with carry in.

## Simpler Drawing



## Programmable Logic Arrays

PLA for short
The dots are really fuses inside a chip
Fuses can be programmed once
Can implement any logic function
Modern "fuses" can be programmed many times
PLAs on hormones are called
Field Programmable Gate Arrays (FPGA)

## PLAS



## Standard Components

Decoders
Multiplexers
ROM

## Decoder



## Multiplexer



## ROM



## Boolean Algebra Laws

Identity Law: $A+0=A, A * 1=A$
Zero \& One Law: $A+1=1, A * 0=0$
Existence of complement: $A+A^{\prime}=1, A^{*} A^{\prime}=0$
Commutative Law: $A+B=B+A, A * B=B * A$
Associative Law: $A+(B+C)=(A+B)+C$

$$
A^{*}\left(B^{*} C\right)=(A * B) * C
$$

Distributive Law: $A^{*}(B+C)=A * B+A * C$

$$
A+(B * C)=(A+B)^{\star}(A+C) \text { (surprising!) }
$$

## De Morgan's Law

$(A+B)^{\prime}=A^{\prime} * B^{\prime}$
$\left(A^{*} B\right)^{\prime}=A^{\prime}+B^{\prime}$
Principle of Duality
AND and OR are symmetric
So is 0 and 1

## Optimization

Two different logic expressions can have exactly the same behavior.

Two different expressions with identical behavior may have different cost of implementation
Choosing the cheapest is optimization May have to satisfy other criteria

Propagation delay, no glitches, etc

## Optimization

$A B+A B^{\prime}$
$=A\left(B+B^{\prime}\right)$
=A*1=A
$A^{\prime} B^{\prime} C+A B C$
$=\left(A^{\prime} B^{\prime}+A B\right) C$
$=\left(\left(A^{\prime} B^{\prime}+A\right)\left(A^{\prime} B^{\prime}+B\right)\right) C$
$\left(B^{\prime}+A\right)\left(A^{\prime}+B\right) C$

## Half Adder

$$
\begin{array}{ll}
S=A^{\prime} B+A B^{\prime} & \\
C=A B & A B \\
& \\
& 00 \\
& 00 \\
& 01 \\
& 10 \\
& 11 \\
& 10 \\
& 11
\end{array}
$$

## Full Adder

$$
\begin{array}{lll} 
& A B C & S C \\
S=A^{\prime} B^{\prime} C+A B^{\prime} C^{\prime}+A^{\prime} B C^{\prime}+A B C & 000 & 00 \\
C^{\prime}+A^{\prime} & 001 & 10 \\
C_{\text {out }}=A B C+A B C '+A B^{\prime} C & 010 & 10 \\
& 011 & 01 \\
& 100 & 10 \\
& 101 & 01 \\
& 110 & 01 \\
& 111 & 11
\end{array}
$$

## Don't Cares

- We use don't cares when we do not care if for a certain combination of input, the output is true or false.
- When this input is illegal and does not appear
- When this input appears only when the output is disabled
- They are very useful when optimizing
- Try it with both outputs and see which one is cheaper


## Verilog

A hardware description language
Can be used to design, optimize and simulate hardware

Started in the mid-80's as a hardware simulation system

Hardware synthesis was added later Its main competitor is VHDL

## What can Verilog do?

Describe a circuit for simulation purposes Many of the Verilog constructs can be synthesizeable.
Allows the designer to specify
Behavior and/or
Structure

## Structure of a Verilog Module

Contains "initial" constructs
Parallel blocks called "always" constructs
Continuous assignments to specify combinational circuits (gates w/o memory) Instances of modules

## Elements of Verilog

Wire: mathematical abstraction of a real wire
Can have 4 possible values!!
True or 1
False or 0
X: unknown (not yet defined, unconnected etc)
$Z$ : high impedance
Electrically disconnected. A smart trick electronics engineers have invented.

## Elements of Verilog

Registers (reg): Are memory elements
The Verilog compiler may map them to actual memory elements (flip-flops)
Same set of possible values

## Elements of Verilog

## Constants

Can be specified as plain constants like 3, 15, 20... Often we want to specify the bit-width of a constant 4'b0011 is 4-bit representation of 3
5 'b00011 is a 5 -bit representation of 3
-4 'b0011 is 4-bit representation of -3 ( 2 's compl.)
4 hF is 4 -bit representation of 15

## Operators in Verilog

$+,-,{ }^{*}, /$ like C
$\&, \mid, \sim, \wedge$ again like $C$
$==,!=,<,>,<=,>=$ like $C$
$\ll, \gg$ like C
con?expr1: expr2 like C

## Operators in Verilog

## But adds to C

Unary \& , |, ^
Apply the operator on all bits of the operand
$\{A, B\}$ the bits of $A$ followed by the bits of $B$
$\{x\{$ const $\}\}$ is $\{$ const,const... $x$ times $\}$

## Combinational Circuits

A network of gates
Directed graph
There should be no cycles (if there are it is not really combinational)
Output determined exclusively by inputs
Implements logic functions

## Combinational Circuits

module h_adder(A, B, S, Cout)
input A, B;
output $S$, Cout;
assign $S=A \wedge B ;$
assign Cout $=A \& B$;
endmodule

## Combinational Circuits

module h_adder (A, B, S, Cout)
input A, B;
output S, Cout;
always @(A, B)
begin
$S<=A \wedge B$;
Cout <= A\&B;
end
endmodule

## Memory elements



## Memory Elements

We can think of memory elements as combinational circuits with feedback

We would rather think of them as little black boxes
Sometimes memory is implemented using other technologies (capacitors for DRAM)

## Combinational Circuits

Module half_adder(A, B,Sum,Carry); input A, B;
output Sum, Carry;
assign Sum = A^B;
assign Carry = A \& \&;
endmodule

## Combinational Circuits

Use the assign keyword
They represent permanent connections
The assign keyword can specify only combinational circuits

Combinational circuits can be specified with the always construct as well
The always construct can also specify sequential circuits

## The always construct

```
Module half_adder(A,B,Sum,Carry)
    input A, B;
    output reg S, C
    always @(A,B) begin
    case ( \(\{\mathrm{A}, \mathrm{B}\}\) )
    2'b00: begin S=0; C=0; end;
    2'b01: begin \(\mathrm{S}=1\); \(\mathrm{C}=0\); end;
    2'b10: begin \(\mathrm{S}=1\); \(\mathrm{C}=0\); end;
    2'b11: begin \(\mathrm{S}=0\); \(\mathrm{C}=1\); end;
    end
endmodule
```


## Combinational with always

Previous example used always to implement a half-adder

Uses blocking assignments
Pretty much the same as $C$ If properly defined, most compilers will not use flip-flops to implement it

If all input signals are on sensitivity list

## Sequential Circuits

Any circuit that contains memory
If it contains memory then it has "state"
If it has state then the state changes, so it goes through a sequence of states
Hence the name sequential.

## Sequential Circuits



## Sequential Circuits

How come signals don't rush around the loop uncontrollably?

This is where the "clock" comes in
It is the same clock you see on the specs of your CPU

With every clock pulse the signal goes around once

These are called synchronous sequential circuits
There are also asynchronous (we do not deal with them)

## Typical Latch



## Still....

Unless the width of the clock pulse is wisely selected...

The signal will travel around more than once These latches are useful in some cases, but not good enough for our current task

## Falling edge trigger FF



## Edge triggered D-Flip-Flop

```
Module DFF(clock,D,Q,Qb)
input clock, D;
output reg Q ;
    output Qb;
    assign \(Q b=\sim\);
    always @(posedge clock)
    Q <= D;
```

endmodule

## Timings

Timing is complex
We use a simplified model
Setup time: time the input to the FF has to be stable before the clock edge
Hold time: time the input has to be stable after the clock edge

## Multibit Wires and Registers

reg [31:0] regA; regA[0] is the LSB; wire [31:0] ALUout; reg [31:0] regfile[0:31]; regfile[0] is the first register in the register file.

## RISC-V ALU

```
module RV_ALU (ALUctl, A, B, ALUOut, Zero);
    input [3:0] ALUctl; input [63:0] A,B;
    output reg [63:0] ALUOut; output Zero;
    assign Zero = (ALUOut==0); //Zero is true if ALUOut is 0
    always @(ALUctl, A, B) begin //reevaluate if these change
        case (ALUctl)
        0: ALUOut <= A & B;
        1: ALUOut <= A | B;
        2: ALUOut <= A + B;
        6: ALUOut <= A - B;
        7: ALUOut <= A < B ? 1 : 0;
        12: ALUOut <= ~(A | B); // result is nor
        default: ALUOut <= 0;
        endcase
    end
endmodule
```


## Register File



## Register File: read



## Register File: write



## Register File: Verilog

module rfile(R1,R2,W,WD,Wctl,RD1,RD2,clock)
input [4:0] R1,R2,W; // Select what to read/write input [63:0] WD; input Wctl, clock;
output [63:0] RD1,RD2;
reg [63:0] RF[31:0];
assign RD1 = RF[R1];
assign RD2 = RF[R2];
always @(posedge clock)
if (Wctl) RF[W] <= WD;
endmodule

## Specifying Gates

Verilog allows the designer to specify individual gates
Can be bulky
Similar syntax can be used for user defined modules

## Half Adder

```
module HA(A,B,S,C)
    input A, B;
    output S, C;
    wire Bn, An, Abn, AnB;
    not N1(An,A);
    not N2(Bn,B);
    and (Abn,A,Bn);
    and (AnB,An,B);
    or (S,ABn,AnB);
    and (C,A,B);
endmodule
```


## Speeding Up Addition

Carry propagation is what slows down addition Sometimes the LSB of input will affect the MSB or the carry out
We design for the worst case scenario
The simpler adders are called ripple adders

## Carry LookAhead

$a 0, a 1, a 2, e t c ; b 0, b 1, b 2$, etc are the inputs c0, c1, c2 are the carries.
$\mathrm{c} 1=\mathrm{b} 0 \mathrm{c} 0+\mathrm{a} 0 \mathrm{c} 0+\mathrm{a} 0 \mathrm{~b} 0$
$\mathrm{c} 1=\mathrm{a} 0 \mathrm{b0}+\mathrm{c} 0(\mathrm{aO}+\mathrm{b} 0)$
c1 $=\mathrm{g} 0+\mathrm{c} 0 \mathrm{p} 0$
$\mathrm{g} 0=\mathrm{a} 0 \mathrm{b0} ; \mathrm{p} 0=\mathrm{a} 0+\mathrm{b} 0$;

## Carry LookAhead

Define

$$
\begin{aligned}
& g_{i}=a_{i} b_{i} \\
& p_{i}=a_{i}+b_{i}
\end{aligned}
$$

Then

$$
c_{i+1}=g_{i}+p_{i} c_{i}
$$

## Carry LookAhead

$\mathrm{c} 1=\mathrm{g} 0+\mathrm{p} 0 \mathrm{c} 0$
c2 = g1 + p1 g0 + p1 p0 c0
c 3 = $\mathrm{g} 2+\mathrm{p} 2 \mathrm{~g} 1+\mathrm{p} 2 \mathrm{p} 1 \mathrm{~g} 0+\mathrm{p} 2 \mathrm{p} 1 \mathrm{p} 0 \mathrm{c} 0$
And...
$\mathrm{c} 4=\mathrm{g} 3+\mathrm{p} 3 \mathrm{~g} 2+\mathrm{p} 3 \mathrm{p} 2 \mathrm{~g} 1+\mathrm{p} 3 \mathrm{p} 2 \mathrm{p} 1 \mathrm{~g} 0+$ p3 p2 p1 p0 c0
$\mathrm{c} 4=\mathrm{G}+\mathrm{P}$ c0
Where $\mathrm{G}=\mathrm{g} 3+\mathrm{p} 3 \mathrm{~g} 2+\mathrm{p} 3 \mathrm{p} 2 \mathrm{~g} 1+\mathrm{p} 3 \mathrm{p} 2 \mathrm{p} 1 \mathrm{~g} 0$
And $P=p 3$ p2 p1 p0

## Delays

- For p0, p1, p2, p3 ............... 1
- For g0, g1, g2, g3 ............... 1
- For c1, c2, c3 ...................... 3
- For c4 .................................. 3 or 4
- For P

2

- For G ................................... 3


## 4-bit Adder



## 16-bit Adder



## 64-bit Adder



