Inference in Propositional Logic

Section 1.6 pages 73-78

- Recall: the reason for studying logic was to formalize derivations and proofs.
- How can we infer facts using logic?
- Simple inference rule (Modus Ponens) : From (a) $p \rightarrow q$ and (b) p is TRUE, we can infer that q is TRUE.
- Many other rules, see page 72.
  - Understanding the rules is crucial, memorizing is not.
  - You should be able to see that the rules make sense and correspond to our intuition about formal reasoning.

Inference Rules (Pg 76)

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Modus Ponens

- Example:
  (a) if these lecture slides are online then you can print them out
  (b) these lecture slides are online

  Can you print out the slides?

- $((p \rightarrow q) \land p) \rightarrow q$ is a TAUTOLOGY.

- Repeated application: From $p \rightarrow q$, $q \rightarrow r$ and p is TRUE, we can infer that r is TRUE.

Other Inference Rules

- Modus Tollens and Disjunctive Syllogism can be seen as alternative forms of Modus Ponens
- Hypothetical syllogism is like the chain rule of implications
- Other rules like "From p is true we can infer $p \vee q$" are very intuitive
- Resolution: From
  (a) $p \vee q$ and (b) $\neg p \vee r$, we can infer : $q \vee r$

  Exercise: check that $((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$ is a TAUTOLOGY.

  Very useful in computer generated proofs.

Correctness of Inference, Proofs

- A Propositional Logic statement is correctly inferred iff it is made using one of the rules of inference listed before
- A proof is correct iff it is a sequence of statements that are either
  - axioms,
  - statements inferred earlier, or
  - statements inferred using one of the rules of inference listed before, and

  the last conclusion is the assertion that needed to be proved.

Terminology: An inferred fact is called a *proposition, lemma* or *theorem* depending on its importance; a special case of a proved statement is sometimes stated as a *corollary*.

Practice on Propositional Logic Inference

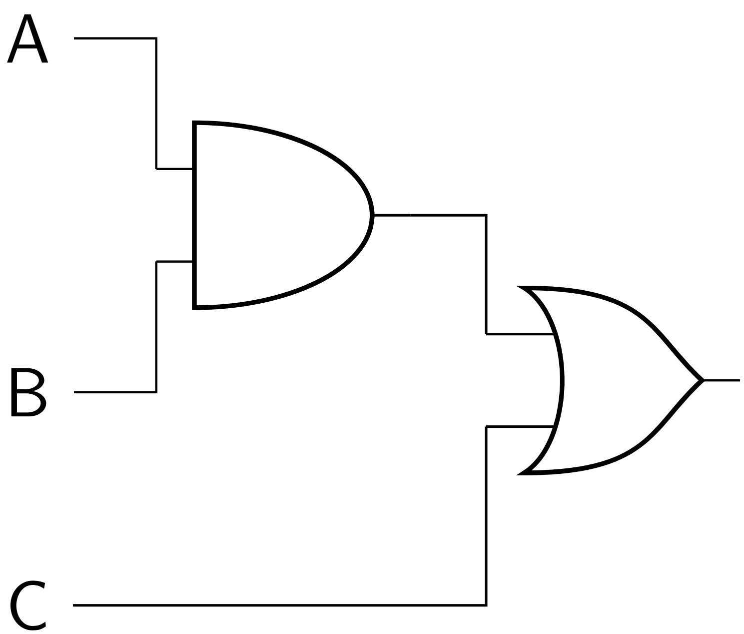
- Q3c, pg 82
  If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool will be closed.

- Q3e, pg 82
  If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburm.

- Q9c, pg 82
  "I am either clever or lucky", ""I am not lucky", "If I am lucky, then I will win the lottery".

Implementing Propositional Logic Statements in Hardware

Typically assume AND, OR, NOT "logic gates", sometimes NAND, NOR, XOR. E.g.,



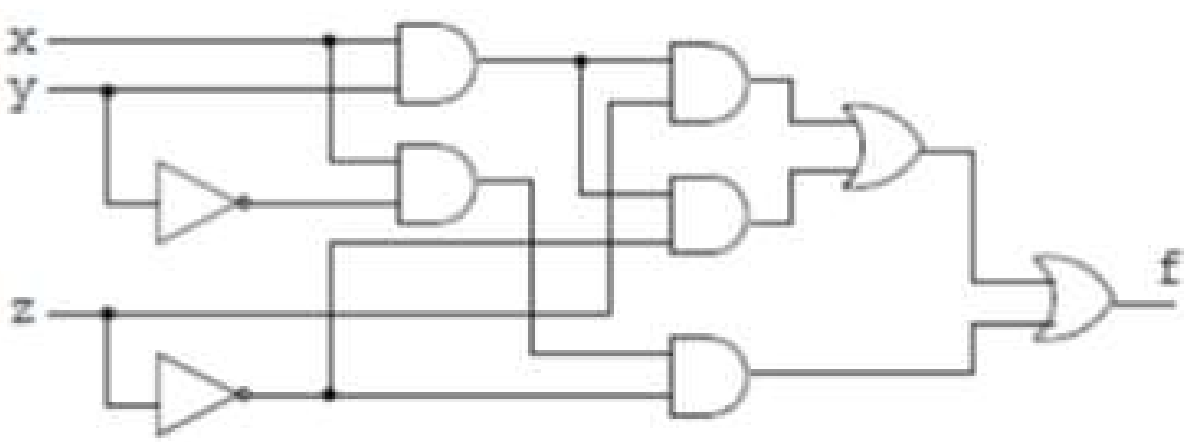
$(A \wedge B) \vee C$

- Often OR and AND are written as $+, \cdot$ respectively

More on Boolean Circuits

- Evaluating Boolean Circuits: Propagate values sequentially by levels, from input to output

- Making a circuit from a propositional logic expression: Disjunctive normal form - ORs of ANDs, e.g., $f(x, y, z) = xyz + xy\overline{z} + x\overline{yz}$



- The function $f(x, y, z)$ simplifies to $x(y + \overline{z})$.

Boolean Circuits from Truth Tables

- Truth Table to DNF:

x	y	z	$f(x, y, z)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

- Disjunction of three terms, one for each 1 entry in the last column
- For each term, put each variable, negated iff it is zero in that row
- So first 1 corresponds to $x\overline{y}\overline{z}$
- So, $f(x, y, z) = xyz + xy\overline{z} + x\overline{y}\overline{z}$

- DNF to circuit: Same as the previous slide

(How) can this circuit be minimized? More advanced courses

Limitations of Propositional Logic

- What can we NOT express using predicates?

  E.g., How do you make a statement about all even integers, like "for all integers x, if $x > 2$ then $x^2 > 4$"?

- A more general language: Predicate logic (Sec 1.4)