EECS 1028 E: Discrete Mathematics for Engineers

Suprakash Datta
Office: LAS 3043

Course page: http://www.eecs.yorku.ca/course/1028
Also on eClass

Administrivia

- Lectures: Mon-Wed-Fri 9:30-10:30 am (VH B, SLH D, LAS A)
- Tests (35%): 3 tests, 15% each (worst test to be scaled to 5%),
- Final Exam (45%), cumulative
- Homework (8%),
- Tutorials (12%),
- Office hours: Mon-Wed 1030 am - 11:30 am or by appointment at LAS 3043

Textbook: Kenneth H. Rosen. Discrete Mathematics and Its Applications, Eighth Edition. McGraw Hill, 2019.

Homework, Grades

- We will be paperless, except for tests, quizzes, final examination.
- All course information online – split across eClass and the public course webpage
- All homework MUST be typed. **You will get a zero** if you submit handwritten solutions. You may use Office, Google Docs, LaTeX, or other packages but all submissions **must be in pdf** format
- We will use Crowdmark for grading. Follow instructions for re-appraisal requests
- All returned work (quizzes, homework, tests) will be on eClass

Tutorials, Quizzes

- Tutorials (2 hours/week) are **mandatory**. Attendance will be taken.
- You get 0.5% for each tutorial attended.
- Every other week, there will be a short quiz (10-15 min), mirrored closely after the tutorial problems covered in the last 2 tutorials. Each quiz is worth 2%, but your attendance points are added to it to a maximum of 2%.
- Missed quizzes and tests cannot be made up. If you have a valid medical reason, the weight will be transferred to the final.
- If you have serious non-medical reasons (having work is not one), talk to me. We will deal with those on an ad hoc basis.

To do well in this class

- Study with pen and paper
- Ask for help early
- Practice, practice, practice ...
- Follow along in class rather than take notes
- Ask questions in class or outside class
- Keep up with the class
- Read the book, not just the slides
- Be timely – HW submitted late will not be graded

Course Objectives - 1

- We will learn to think differently!
- Ask why instead of how
- Reason about statements mathematically

Course Objectives - 2

We will focus on two major goals:

- Basic tools and techniques in discrete mathematics
    - Set Theory, Functions and Relations
    - Propositional and Predicate logic
    - Sequences and Series
    - Induction, Recursion
    - Simple Combinatorics
    - Introductory Graph Theory
- Precise and Rigorous Mathematical Reasoning
    - Writing proofs

Mathematical Reasoning

- Why Mathematics?
  – Mathematics as a precise language
    - Precision in definitions
    - Precision in statements
- Motivation (for EECS)
    - Specification (description, modeling)
    - Reasoning (Making precise, rigorous claims)
- Procedure
    - Axioms
    - Inference
    - Facts/Theorems

Examples of Reasoning about Problems

- $0.9999999999999999\ldots = 1$?
- There exists integers a, b, c that satisfy the equation $a^2 + b^2 = c^2$
- There exists integers a, b, c that satisfy the equation $a^4 + b^4 = c^4$
- There are as many integers as there are rational numbers
- The program that I wrote never hangs (i.e. always terminates)...
- The program that I wrote works correctly for all valid inputs
- There does not exist an algorithm to check if a given program never hangs

Proofs and Similar Structures

- Backing up statements with reasons
- Providing detailed explanations (Amazon, Netflix,...)
- Understanding the basis of eCommerce, Bitcoin, ...
- Evidence in data mining systems

Intuitive Proofs

- What?
- Why?
- When?
- How much detail?

Review of Fundamentals

- Sets
- Number Systems
- Basic algebra

Set Theory Fundamentals

- Unordered collection of elements, e.g.,
  - Single digit integers
  - Non-negative integers
  - Faces of a die
  - Sides of a coin
  - Students who finished EECS1028, W 2018
- Two key aspects of sets:
  - No duplicates
  - No inherent ordering of elements

Set Theory Fundamentals - 2

- $\mathbb{N}$: the set of natural numbers, $\mathbb{R}$: the set of real numbers
- Membership
  Notation: $a \in A, b \notin A$
- Ordered pairs
  Notation: $(a, b), a \in A, b \in B$
- Equality of sets

Describing Sets

- English description
  - The set of natural numbers between 5 and 8 (inclusive).
  - The set of all students who finished EECS1028 M, W 2018
- Enumeration
  - $S = \{1, 2, 3\}$
  - $S = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$
- Set builder-notation
  - $S = \{x \in \mathbb{N} | x > 3\}$
  - $S = \{(x, y) | x, y \in \{0, 1\}\}$

More on Sets

Special sets

- Universal set U
- Empty set ϕ (How many elements?)

Sets vs Sets of sets

- $\{1,2\}$ vs $\{\{1,\},\{2\}\}$
- $\{\}$ vs $\{\{\}\} = \{\phi\}$

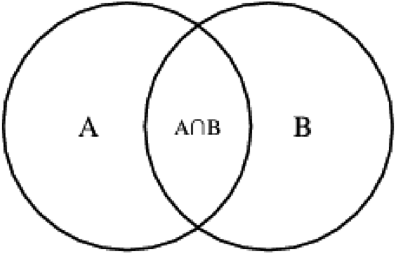
Note:

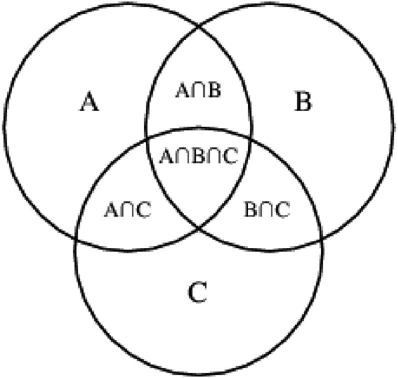
1. Connection with data types (e.g., in Java)
2. The elements of a set can be sets, pairs of elements, pairs of pairs, triples,. . . !!

Set Operations

- Subsets: $A \subseteq B$: each element of A is in B
- Union: $A \cup B = \{x | (x \in A) \text{ or } (x \in B)\}$
- Intersection: $A \cap B = \{x | (x \in A) \text{ and } (x \in B)\}$
- Difference: $A - B = \{x | (x \in A) \text{ and } (x \notin B)\}$
- Complement: A^c or $\overline{A} = \{x | x \notin A\} = U - A$
- Cartesian product: $AxB = \{(a, b) | a \in A, b \in B\}$
    - "Set of ordered pairs"
    - $\mathbb{R}x\mathbb{R} = \{(x, y) | x \in \mathbb{R}, y \in \mathbb{R}\}$, "Coordinate plane" or "the real plane" Sometimes called $\mathbb{R}^2$.

Venn Diagrams





More on Sets

- Cardinality - number of (distinct) elements
- Finite set - cardinality some finite integer n
- Infinite set - a set that is not finite
- Power set - Set of all subsets; Notation $\mathcal{P}(S) = \{A | A \subseteq S\}$, sometimes written 2^S

Laws on Set operations

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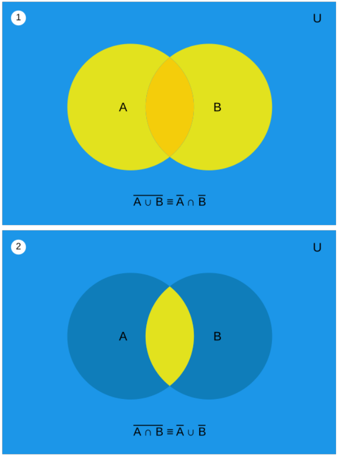
- Associative laws
  $(A \cup B) \cup C = A \cup (B \cup C)$
  $(A \cap B) \cap C = A \cap (B \cap C)$

- Distributive laws:
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- De Morgan's laws
  $\overline{A \cup B} = \overline{A} \cap \overline{B}$
  $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Proofs of Laws of Set Operations

Proofs can be done with Venn diagrams.



Number Systems

- Natural numbers, $\mathbb{N} : \{1, 2, 3, ...\}$.
- Whole numbers, $\mathbb{W} : \{0, 1, 2, 3, ...\}$.
- Integers, $\mathbb{Z} : \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$
  Notation: $\mathbb{Z}^+ =$ positive integers $= \mathbb{N}$
- Real numbers, $\mathbb{R}$
  Notation: $\mathbb{R}^+ =$ positive reals
- Complex numbers, $\mathbb{C} = \{x + iy | x, y \in \mathbb{R}, i^2 = -1\}$.
- Co-ordinates on the plane, $\mathbb{R}^2$.
- Rational numbers, $\mathbb{Q} = \{\frac{m}{n} | m, n \in \mathbb{Z}, n \neq 0\}$.
- Irrational numbers, $\mathbb{R} - \mathbb{Q}$: all real numbers that are not rational. Examples: $\pi, e, \sqrt{2}$.

Number Systems - Questions

- How do we know $\pi, e, \sqrt{2}$ are not rational?
- How are real numbers represented on a computer?
- Do all rational numbers have finite decimal representations? Counterexample: $1/3$
- If a number has an infinite decimal representation, can we conclude it is irrational?

Basic Algebra (please review)

- Operations with exponents: Theorem 1, pg A-7
  - $b^x * b^y = b^{x+y}$
  - $b^x / b^y = b^{x-y}$
  - $(b^x)^y = b^{xy}$
- Logarithms: Theorem 2, pg A-8
  - Logarithm of products, powers
  - Change of bases
- Operations with polynomials
- Solving linear and quadratic equations

Check your understanding

Let $A = \{0, 1\}$.

1. What is $\emptyset - A$?
2. What is $A \times \emptyset$?
3. What is $\emptyset \times \emptyset$?
4. What is $\mathcal{P}(\emptyset)$?

Answer key:
1. $\emptyset$
2. $\emptyset$
3. $\emptyset$
4. $\{\emptyset\}$

A Simple Proof

Claim: $0.999999999999999\ldots = 1$.
Proof: Let $x = 0.999999999999999\ldots$. Therefore,

$$\begin{aligned} 10x &= 9.999999999999999\ldots \\ 10x - x &= 9 \\ x &= 1 \end{aligned}$$

Note: You may recognize that $x = \frac{9}{10} + \frac{9}{100} + \ldots$. This is a geometric series and we computed the sum above.

Another Example

Claim: Let $n \in \mathbb{N}$. If n^2 is even, then n is even.
Proof: Suppose this is false. Then n is odd. But $n^2 = n * n$ must be odd because the product of 2 odd numbers is odd. That is a contradiction!
Why? We assumed that n^2 is even but now it turns out that n^2 is odd.

Something went wrong! Our algebra was correct, so our original assumption (that n is odd) is incorrect.

Therefore n cannot be odd, and so it must be even.

This is a proof by contradiction

Note: There are several other ways to prove this.

Proof that $\sqrt{2}$ is not rational

Proof by contradiction:
Let's suppose the statement is false; i.e., $\sqrt{2}$ is a rational number. Then
$\sqrt{2} = a/b$ where a, b are integers, $b \neq 0$.
We ALSO assume that a/b is simplified to lowest terms, i.e., $gcd(a, b) = 1$. So,

$$\begin{aligned} \sqrt{2} &= a/b \\ 2 &= a^2/b^2, \text{ squaring} \\ a^2 &= 2b^2 \end{aligned}$$

So a^2 is even implying that a is also even (from the last slide)
Since a is even, then $a = 2k$ for some integer k.

Proof that $\sqrt{2}$ is not rational - Continued

Substituting $a = 2k$ we get:

$$\begin{aligned} a^2 &= 2b^2 \\ 4k^2 &= 2b^2 \\ b^2 &= 2k^2 \end{aligned}$$

So b^2 is even, implying b is even (again from the last claim we proved).
We assumed that $gcd(a, b) = 1$ but now it turns out that a, b are both even, so $gcd(a, b) \geq 2$. That is a contradiction.

So our original assumption (that $\sqrt{2}$ is rational) is incorrect.

Therefore $\sqrt{2}$ cannot be rational.