Homework Assignment #7
Due: November 24, 2023 at 7:00 p.m.

1. Let $L_1 = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k : i, j, k \in \mathbb{N} \text{ and either } i = j \text{ or } j = k\}$.

[4] **(a)** Give a context-free grammar for L_1.

You do not have to prove that your answer is correct, but you should give a precise description of the strings that can be generated by each variable in your grammar.

[2] **(b)** Show that your grammar is ambiguous.
(This statement will be true if your solution in part (a) is correct.)

[5] **2.** Let $L_2 = \{x\mathtt{\#}y : x, y \in \{\mathtt{a}, \mathtt{b}\}^* \text{ and the number of } \mathtt{a}\text{'s in } x \text{ is the same as the number of } \mathtt{b}\text{'s in } y\}$. Give a detailed proof that the following context-free grammar generates every string in L_2.

$$
\begin{aligned}
S &\to \mathtt{\#} \\
S &\to \mathtt{b}S \\
S &\to S\mathtt{a} \\
S &\to \mathtt{a}S\mathtt{b}
\end{aligned}
$$