

Homework Assignment #8
Due: November 10, 2023 at 7:00 p.m.

1. Let $TIME = \{\langle M, t \rangle : M \text{ is a TM and } t \in \mathbb{N} \text{ and } M \text{ runs for at least } t \text{ steps on input } \varepsilon\}$. Show that $TIME$ is decidable.

Remark: For the sake of precision, we will specify exactly how the encoding $\langle M, t \rangle$ is done using the alphabet $\{<, >, 0, 1, \#\}$. $\langle M, t \rangle$ is a string of the form $<x\#y>$, where x is a string that encodes the TM M in binary (M is first encoded in YUTMFF, which is an ASCII file, which in turn is encoded as a binary string by converting each ASCII character into an 8-bit binary string) and y is the standard binary representation of the natural number t .

2. Let $HALT\varepsilon = \{\langle M \rangle : M \text{ is a TM that halts on input } \varepsilon\}$.

(a) Show that $HALT\varepsilon$ is not decidable.

(b) Show that $\overline{HALT\varepsilon}$ is not recognizable.

3. Recall the definition of $SUB(L)$ from Assignment 7. Are decidable languages closed under SUB ? In other words, is it true that for all languages L , if L is decidable then $SUB(L)$ is decidable? Prove your answer is correct.

Hint: Your answer(s) to questions 1 and/or 2 might be useful here.