

Homework Assignment #4

Due: October 6, 2023 at 7:00 p.m.

For full marks, your answers should be as simple as possible.

1. Consider the alphabet $\left\{ \begin{smallmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{smallmatrix} \right\}$. We shall use strings in this alphabet to describe two integers: one using the top row of bits and one using the bottom row. Each integer is represented in binary. For example, to represent the two integers 13 and 7 (whose binary representations are 1101 and 111), we would use the string $\begin{smallmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{smallmatrix}$: the top row is 1101, and the bottom row is 111 (the extra 0 at the beginning of the bottom row is just padding to make the two rows the same length).
- [3] (a) Let *LESS* be the language of all strings where the integer represented in the top row is less than the integer represented by the bottom row. For example, the string $\begin{smallmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{smallmatrix}$ is not in *LESS*, since 13 is not less than 7. Give a regular expression for the language *LESS*. Briefly explain how you came up with your answer.
- [3] (b) Let *PLUS1* be the language of all strings where the integer represented in the top row plus one is equal to the integer represented in the bottom row. For example, the string $\begin{smallmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{smallmatrix}$ is in *PLUS1* because the top row represents the integer 5 and the bottom row represents the integer 6 and $5 + 1 = 6$. Give a regular expression for the language *PLUS1*. Briefly explain how you came up with your answer.

2. For this question, the alphabet is $\{a, b\}$. Consider the regular expression

$$R = (aa^*bb^*aa^* \cup bb^*aa^*bb^*)(a \cup b)^*.$$

- [1] (a) List four strings generated by *R*.
- [1] (b) List four strings *not* generated by *R*.
- [2] (c) *R* describes the set of all strings that have both of the strings _____ and _____ as substrings. Fill in the blanks and briefly explain why your answer is correct.

Optional programming exercise (just for fun; do not hand in): Write a java programme that reads a description of a finite automaton (in the same format as described in assignment #2), and outputs some string that is accepted by the automaton. If no such string exists, your programme should output “NO STRING ACCEPTED”.