

Homework Assignment #6

Due: March 13, 2023 at 10:00 p.m.

1. Consider an asynchronous system of n processes, where processes may experience halting failures.

We can define the **1-2-Counter** type as follows. The state of the object stores a natural number. It provides three operations: **READ** returns the state of the object without changing it, **INC** increases the state of the object by 1 and returns ack, and **INC-BY-2** increases the state of the object by 2 and returns ack.

- [3] (a) Here is a proposed implementation of a **1-2-Counter** for n processes from n shared **read/write registers**, $A[1], \dots, A[n]$. Process i would execute the following code to perform an operation on the **1-2-counter**. (Here, x and v are local variables of the process performing the operation.)

```

1: function READ
2:    $v \leftarrow 0$ 
3:   for  $j \leftarrow 1$  to  $n$  do
4:      $v \leftarrow v + A[j]$ 
5:   end for return  $v$ 
6: end function
7:
8: function INC
9:    $x \leftarrow A[i]$ 
10:   $A[i] \leftarrow x + 1$ 
11: end function
12:
13: function INC-BY-2
14:    $x \leftarrow A[i]$ 
15:    $A[i] \leftarrow x + 2$ 
16: end function

```

▷ This is a read of **register** $A[j]$

▷ This is a read of **register** $A[i]$

▷ This is a write to **register** $A[i]$

▷ This is a read of **register** $A[i]$

▷ This is a write to **register** $A[i]$

Prove this is *not* a linearizable implementation.

- [3] (b) Is there a deterministic, wait-free, linearizable implementation of a **1-2-counter** from read/write registers? Prove your answer is correct.
- [4] (c) Is there a deterministic, lock-free, linearizable, *anonymous* implementation of a **1-2-counter** from registers? Here, anonymous means that processes have no unique ids and all processes have identical code for each operation. Prove your answer is correct.

Hint: think about what happens when two processes trying to do the same operation run at exactly the same speed.