York University EECS 6117 January 17, 2023

Homework Assignment #1
Due: January 25, 2023 at 5:00 p.m.

1. Recall the Two Generals problem from class. We shall consider a more general problem, where there are n generals instead of two, and we want a majority of them to agree on a time. The generals have synchronized watches and each general begins with an initial preference for what time to attack. Generals can communicate only by messengers. Each general may send a messenger to any other general. However, messengers may be captured. (For simplicity, you may assume that any messenger that is not captured arrives exactly 10 minutes after setting out.) After communicating with the other generals according to some algorithm, each general must decide what time to attack. We assume that each execution uses a finite number of messengers, but there is no bound on how many messengers can be used. The algorithm should satisfy the following properties.

- Majority Agreement: In *every* execution of the algorithm, at least $\lceil n/2 \rceil$ generals' decisions are identical.
- Validity: If all the generals start with the same preference and no messengers are captured, then all generals should decide on that common initial preference.

Note: $\lceil x \rceil$ denotes the smallest integer that is greater than or equal to x.

(a) Give a *simple* algorithm that the n generals can follow to solve this problem. Explain why your algorithm satisfies majority agreement, validity and termination.
Hint: Your algorithm may assign different sets of instructions to different generals.

(b) When $n = 4$, your algorithm in part (a) guarantees that at least two generals will agree. We want to show there is no algorithm that solves the problem for $n = 4$ if the Majority Agreement property is strengthened to require that (in every execution) at least *three* generals output the same time. Assume there exists an algorithm A that solves the problem with this strengthened condition. The goal is to derive a contradiction.

Let α be the execution of A where all generals have the initial preference of noon, and all messengers arrive. Since generals must eventually decide, there can only be a finite number of messages sent during α. Let k be the number of messages sent. For $0 \leq i \leq k$, let α_i be the execution of A where all processes have input noon, the first i messengers sent arrive, and all other messengers are captured. (If the algorithm specifies that two messengers are supposed to be sent at exactly the same time, break ties between them in some arbitrary way.)

(i) Argue that at most one general whose output in α_i is different from his output in α_{i+1} (for $0 \leq i < k$).

(ii) Argue that, in α_0, at least three generals must output noon.

(iii) By an argument analogous to part (i) and (ii), in the execution β_0 where all generals have initial preference 11:00, and all messengers are captured, there must be at least three generals that output 11:00. Derive a contradiction.