York University

EECS 3101Z

Homework Assignment #1 Due: January 23, 2023 at 7:00 p.m.

- 1. Let A[1..n] be an array of integers, where n > 1 and A[1] = 1 and A[n] > n. We want to find a place in the array where the difference between one element to the next is greater than 1.
- [2] (a) Prove that for any $i \leq j \leq k$, if A[k] A[i] > k i then either A[k] A[j] > k j or A[j] A[i] > j i.
- [6] (b) Fill in the blanks in the following algorithm so that it runs correctly in $O(\log n)$ time.

Hint: write the invariant first, and then fill in the rest of the blanks.

- 1: preconditions: A[1..n] is array of integers, n > 1, A[1] = 1, A[n] > n2: $lo \leftarrow 1$ 3: $hi \leftarrow n$ 4: **loop** invariant: _____ 5: exit when ______ $mid \leftarrow \lfloor \frac{lo+hi}{2} \rfloor$ if ______ then _____ 6: 7:8: 9: else 10: $hi \leftarrow_-$ 11: end if 12:13: end loop 14: **postcondition:** A[lo + 1] > A[lo] + 1
- [5] (c) Prove that your invariant is true at the start of each iteration of the loop.
- [3] (d) Prove that the loop terminates.
- [1] (e) Prove that when the loop terminates, the postcondition is satisfied.
- [1] (f) Give a good upper bound on the worst-case running time of the algorithm. Use big-O notation to give your bound in terms of n. You do not have to prove your answer is correct.