York University EECS 4101/5101 September 14, 2022

Homework Assignment #1
Due: September 21, 2022 at 5:00 p.m.

Before working on this assignment, you should complete and submit (via eclass) the declaration posted on the course web page. Your solutions should also be submitted via eclass.

1. Consider a binary search tree T that contains the integer keys $1, \ldots, n$. We shall measure the time to perform a Search by counting the number of nodes of T the Search must visit before outputting an answer. Let $W_{search}(T)$ be the worst-case time to search for any key in T:

$$W_{search}(T) = \max_{1 \le k \le n} (\text{number of nodes visited by } \textsc{Search}(k) \text{ in } T).$$

Let $A_{search}(T)$ be the average time to search for a key (chosen uniformly at random) in T:

$$A_{search}(T) = \frac{1}{n} \sum_{k=1}^{n} (\text{number of nodes visited by } \textsc{Search}(k) \text{ in } T).$$

Prove or disprove the following statement.

There is a constant c such that, for all n and for all binary search trees T containing keys $1, \ldots, n$, if $A_{search}(T) \le 2 \log_2 n$ then $W_{search}(T) \le c \log_2 n$.

2. Consider the implementation of the counter that we described in class: the value is stored in binary representation in a linked list of bits. The list is accessed by a pointer to the least significant bit. Now suppose the counter supports two additional operations Double that multiplies the value of the counter by 2 and Halve that divides the value by 2 (rounding up).

- **(a)** *Briefly* describe how you would implement the Double and Halve operations.
- **(b)** Use Θ notation to give a tight upper bound on the worst-case time required for a sequence of m Increment, Double and Halve operations, if the counter initially has the value 0. Prove your answer is correct (using any method you like).

3. Now consider a version of the counter that supports three operations: Double, which multiplies the value by 2, Increment, which increases the value by 1, and Decrement, which decreases the value by 1. (If the value is 0, Decrement should have no effect.)

- **(a)** *Briefly* describe how you would implement Decrement, including how you would handle the case when the value is 0.
- **(b)** Use Θ notation to give a tight upper bound on the worst-case time required for a sequence of m Increment, Decrement and Double operations, if the counter initially has the value 0. Prove your answer is correct (using any method you like).