Example 1

Consider the knowledge base on slide 33 of the B&L lecture notes

\[ S = \{ \text{On}(a, b), \text{On}(b, c), \text{Green}(a), \neg \text{Green}(c) \} \]

Does \( S \models \text{Green}(b) \)?

No. Prove it.

How?

Example 1

Prove that \( S \not\models \text{Green}(b) \) by giving an interpretation \( I = \langle D, I \rangle \) and showing that \( I \models S \cup \{ \neg \text{Green}(b) \} \).
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Prove that $S \not\models \text{Green}(b)$ by giving an interpretation $I = \langle D, I \rangle$ and showing that $I \models S \cup \{\neg \text{Green}(b)\}$

Let $D = \{a, b, c\}$,
$I(a) = a_i, I(b) = b_i, I(c) = c_i$
$I(\text{Green}) = \{a_i\},$ and
$I(\text{On}) = \{\langle a, b_i \rangle, \langle b, c_i \rangle\}$.

It is easy to show that $I \models S \cup \{\neg \text{Green}(b)\}$.

There are many other such interpretations.

Example 2

Does $S \models \neg \exists x \text{On}(c, x)$?

No.

To prove it, we can use the same interpretation as above except with
$I(\text{Green}) = \{a_i, b_i\}$.

Then $I \models S \cup \{\text{Green}(b)\}$.
Example 2

Let $S' = S \cup \{ \forall x \forall y. \text{On}(x, y) \supset (x = a \land y = b) \lor (x = b \land y = c) \}$. Does $S' \models \neg \exists x \text{On}(c, x)$?

No, because nothing ensures that $c$ is not equal to $a$ or $b$ (e.g., we can have $I(c) = a_i$).

Example 2

Let $S'' = S' \cup \{ a \neq b \land b \neq c \land a \neq c \}$. Does $S'' \models \neg \exists x \text{On}(c, x)$?
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Let $S'' = S' \cup \{ a \neq b \land b \neq c \land a \neq c \}$. Does $S'' \models \neg \exists x \text{On}(c, x)$? Yes.

Proof:
Take an arbitrary interpretation $I$ and assume that $I \models S''$.

$I \models \forall x \forall y. \text{On}(x, y) \supset (x = a \land y = b) \lor (x = b \land y = c)$

$I \models c \neq a$

$I \models c \neq b$

Thus $I \models \neg \exists x \text{On}(c, x)$?

Therefore $S'' \models \neg \exists x \text{On}(c, x)$.

Exercise 1

Write a sentence in FOL that represents the following knowledge:

Every professor who is not member of any committee is happy.

$\forall x. \text{Prof}(x) \land \neg \exists y. \text{Member}(x, y) \supset \text{Happy}(x)$

or

$\forall x. \text{Prof}(x) \land \neg \exists y. \text{Committee}(y) \land \text{Member}(x, y) \supset \text{Happy}(x)$

or

$\forall x. \text{Prof}(x) \land (\forall y. \text{Committee}(y) \supset \neg \text{Member}(x, y)) \supset \text{Happy}(x)$

Exercise 2

Write a sentence in FOL that represents the following knowledge:

There is a team member who likes every team member except Mary.

$\forall x. \text{Prof}(x) \land \neg \exists y. \text{Member}(x, y) \supset \text{Happy}(x)$

or

$\forall x. \text{Prof}(x) \land \neg \exists y. \text{Committee}(y) \land \text{Member}(x, y) \supset \text{Happy}(x)$

or

$\forall x. \text{Prof}(x) \land (\forall y. \text{Committee}(y) \supset \neg \text{Member}(x, y)) \supset \text{Happy}(x)$
Exercise 2

Write a sentence in FOL that represents the following knowledge:

There is a team member who likes every team member except Mary.

$$\exists x. \text{Member}(x) \land \forall y. \text{Member}(y) \land \neg y = \text{mary} \supset \text{Likes}(x, y)$$