

EECS 4401/5326 Winter 2021

Week 5 — Additional Examples

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Exercise 1

Use the tableau method for \mathcal{ALC} described in Baader and Sattler's paper to check whether the following concept is satisfiable/consistent. Show the steps and rules that are used. If the concept is satisfiable give the model(s) (satisfying interpretation(s)) obtained by the method.

$$((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcup (\forall R.\neg B)))$$

Solution

Let

$$C_0 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcup (\forall R.\neg B)))$$

$$C_1 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B))$$

$$C_2 = (\forall R.((\forall R.\neg A) \sqcup (\forall R.\neg B)))$$

Start with

$$x_o \bullet \{C_0\}$$

Apply \rightarrow_{\sqcap} to get

$$x_o \bullet \{C_0, C_1, C_2\}$$

Apply \rightarrow_{\cap} again to get

$$x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\}$$

Apply \rightarrow_{\exists} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B\} \end{array}$$

Apply \rightarrow_{\forall} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B, \exists R.A\} \end{array}$$

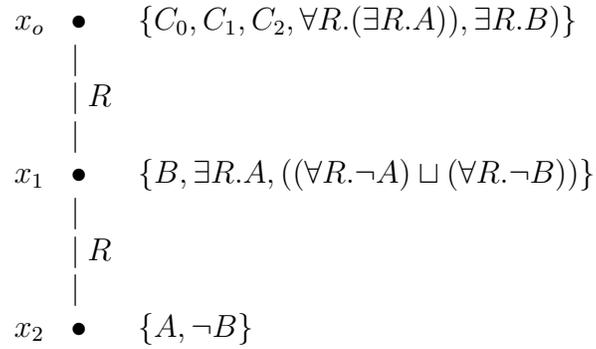
Apply \rightarrow_{\forall} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B, \exists R.A, ((\forall R.\neg A) \sqcup (\forall R.\neg B))\} \end{array}$$

Apply \rightarrow_{\exists} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B, \exists R.A, ((\forall R.\neg A) \sqcup (\forall R.\neg B))\} \\ | \\ R \\ | \\ x_2 \bullet \{A\} \end{array}$$

Apply \rightarrow_{\sqcup} again to get



No clash and no further rules apply so C_0 is satisfiable.

A model I is $\Delta^I = \{x_0, x_1, x_2\}$ and $A^I = \{x_2\}$ and $B^I = \{x_1\}$.

Exercise 2

Use the tableau method for \mathcal{ALC} described in Baader and Sattler's paper to check whether the following concept is satisfiable/consistent. Show the steps and rules that are used. If the concept is satisfiable give the model(s) (satisfying interpretation(s)) obtained by the method.

$$((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcap (\forall R.\neg B)))$$

Solution

Let

$$C_0 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcap (\forall R.\neg B)))$$

$$C_1 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B))$$

$$C_2 = (\forall R.((\forall R.\neg A) \sqcap (\forall R.\neg B)))$$

Start with

$$x_0 \bullet \quad \{C_0\}$$

Apply \rightarrow_{\sqcap} to get

$$x_0 \bullet \quad \{C_0, C_1, C_2\}$$

Apply \rightarrow_{\sqcap} again to get

$$x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\}$$

Apply \rightarrow_{\exists} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B\} \end{array}$$

Apply \rightarrow_{\forall} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B, \exists R.A\} \end{array}$$

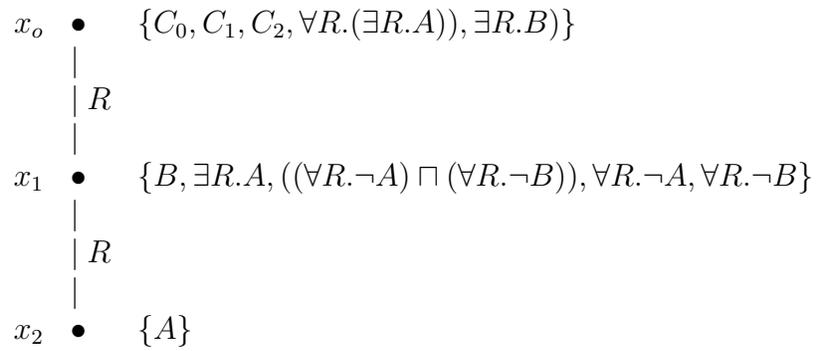
Apply \rightarrow_{\forall} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B, \exists R.A, ((\forall R.\neg A) \sqcup (\forall R.\neg B))\} \end{array}$$

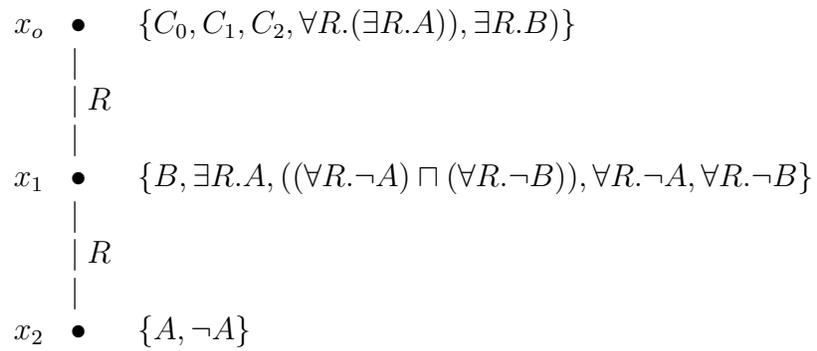
Apply \rightarrow_{\exists} to get

$$\begin{array}{c} x_o \bullet \{C_0, C_1, C_2, \forall R.(\exists R.A), \exists R.B\} \\ | \\ R \\ | \\ x_1 \bullet \{B, \exists R.A, ((\forall R.\neg A) \sqcap (\forall R.\neg B))\} \\ | \\ R \\ | \\ x_2 \bullet \{A\} \end{array}$$

Apply \rightarrow_{\sqcap} again to get



Apply \rightarrow_{\forall} to get



So we have a clash!

There are no alternatives/other ways to apply the or rule, so C_0 is unsatisfiable.