EECS 4401/5326 Winter 2021
Week 5 — Additional Examples

Yves Lespérance
Dept. of Electrical Engineering and Computer Science
York University

February 9, 2021

Exercise 1

Use the tableau method for $\mathcal{ALC}$ described in Baader and Sattler's paper to check whether the following concept is satisfiable/consistent. Show the steps and rules that are used. If the concept is satisfiable give the model(s) (satisfying interpretation(s)) obtained by the method.

$$((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcup (\forall R.\neg B)))$$

Solution

Let

$$C_0 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcup (\forall R.\neg B)))$$
$$C_1 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B))$$
$$C_2 = (\forall R.((\forall R.\neg A) \sqcup (\forall R.\neg B)))$$

Start with

$$x_o \quad \bullet \quad \{C_0\}$$

Apply $\rightarrow_\sqcap$ to get

$$x_o \quad \bullet \quad \{C_0, C_1, C_2\}$$

Apply $\rightarrow_{\sqcap}$ again to get

$$x_o \quad \bullet \quad \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\}$$

Apply $\rightarrow_{\exists}$ to get

$$\begin{array}{lcl} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B\} \end{array}$$

Apply $\rightarrow_{\forall}$ to get

$$\begin{array}{lcl} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B, \exists R.A\} \end{array}$$

Apply $\rightarrow_{\forall}$ to get

$$\begin{array}{lcl} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B, \exists R.A, ((\forall R.\neg A) \sqcup (\forall R.\neg B))\} \end{array}$$

Apply $\rightarrow_{\exists}$ to get

$$\begin{array}{lcl} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B, \exists R.A, ((\forall R.\neg A) \sqcup (\forall R.\neg B))\} \\ & \mid R & \\ x_2 & \bullet & \{A\} \end{array}$$

Apply $\rightarrow_{\sqcup}$ again to get

$$
\begin{array}{lcl}
x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\
 & \Big| R & \\
x_1 & \bullet & \{B, \exists R.A, ((\forall R.\neg A) \sqcup (\forall R.\neg B))\} \\
 & \Big| R & \\
x_2 & \bullet & \{A, \neg B\}
\end{array}
$$

No clash and no further rules apply so C_0 is satisfiable.
A model I is $\Delta^I = \{x_0, x_1, x_2\}$ and $A^I = \{x_2\}$ and $B^I = \{x_1\}$.

Exercise 2

Use the tableau method for $\mathcal{ALC}$ described in Baader and Sattler's paper to check whether the following concept is satisfiable/consistent. Show the steps and rules that are used. If the concept is satisfiable give the model(s) (satisfying interpretation(s)) obtained by the method.

$$((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcap (\forall R.\neg B)))$$

Solution

Let

$$
\begin{array}{l}
C_0 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \sqcap (\forall R.((\forall R.\neg A) \sqcap (\forall R.\neg B))) \\
C_1 = ((\forall R.(\exists R.A)) \sqcap (\exists R.B)) \\
C_2 = (\forall R.((\forall R.\neg A) \sqcap (\forall R.\neg B)))
\end{array}
$$

Start with

$$x_o \quad \bullet \quad \{C_0\}$$

Apply $\rightarrow_\sqcap$ to get

$$x_o \quad \bullet \quad \{C_0, C_1, C_2\}$$

Apply $\rightarrow_\sqcap$ again to get

$$x_o \quad \bullet \quad \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\}$$

Apply $\rightarrow_{\exists}$ to get

$$\begin{array}{lll} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B\} \end{array}$$

Apply $\rightarrow_{\forall}$ to get

$$\begin{array}{lll} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B, \exists R.A\} \end{array}$$

Apply $\rightarrow_{\forall}$ to get

$$\begin{array}{lll} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B, \exists R.A, ((\forall R.\neg A) \sqcup (\forall R.\neg B))\} \end{array}$$

Apply $\rightarrow_{\exists}$ to get

$$\begin{array}{lll} x_o & \bullet & \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\} \\ & \mid R & \\ x_1 & \bullet & \{B, \exists R.A, ((\forall R.\neg A) \sqcap (\forall R.\neg B))\} \\ & \mid R & \\ x_2 & \bullet & \{A\} \end{array}$$

Apply $\rightarrow_{\sqcap}$ again to get

$x_o \bullet \quad \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\}$

$\mid R$

$x_1 \bullet \quad \{B, \exists R.A, ((\forall R.\neg A) \sqcap (\forall R.\neg B)), \forall R.\neg A, \forall R.\neg B\}$

$\mid R$

$x_2 \bullet \quad \{A\}$

Apply $\rightarrow_\forall$ to get

$x_o \bullet \quad \{C_0, C_1, C_2, \forall R.(\exists R.A)), \exists R.B)\}$

$\mid R$

$x_1 \bullet \quad \{B, \exists R.A, ((\forall R.\neg A) \sqcap (\forall R.\neg B)), \forall R.\neg A, \forall R.\neg B\}$

$\mid R$

$x_2 \bullet \quad \{A, \neg A\}$

So we have a clash!
There are no alternatives/other ways to apply the or rule, so C_0 is unsatisfiable.