Assignment 1

Total marks: 70.

Out: February 4
Due: February 22 at 10:00am

Note: Your report for this assignment should be the result of your own individual work. Take care to avoid plagiarism ("copying"). You may discuss the problems with other students, but do not take written notes during these discussions, and do not share your written solutions.

1. [30 points] Consider the following piece of knowledge:

   > Tony, Mike, and John belong to the Alpine Club. Every member of the Alpine Club who is not a skier is a mountain climber. Mountain climbers do not like rain, and anyone who does not like snow is not a skier. Mike dislikes whatever Tony likes and likes whatever Tony dislikes. Tony likes rain and snow.

   a) Write some sentences in first-order logic that *represent* the knowledge given above. Use the notation in Brachman and Levesque's book for this question. Also provide a glossary where you indicate the intended meaning of your predicate, function, and constant symbols in English.

   b) *Prove* that the given sentences *entail* that there is a member of the Alpine Clib who is a mountain climber but not a skier.

   c) Suppose that we had been told that Mike likes whatever Tony dislikes, but we had not been told that Mike dislikes whatever Tony likes. *Prove* that the resulting set of sentences *no longer entail* that there is a member of the Alpine Club who is a mountain climber but not a skier.

   For this question, your proofs should use the definition of entailment and refer to interpretations; do not use resolution.

2. [20 points]

   a) Show that the following sentence is *valid* in the description logic of Brachman and Levesque Chapter 9:

   $$[AND \ [ALL \ r_1 \ [AND \ d_1 \ d_2 \ d_3]] \ [FILLS \ r_1 \ c_1]]$$
   $$\sqsubseteq [AND \ [ALL \ r_1 \ d_1] \ [EXISTS \ 1 \ r_1]]$$

b) Show that the following sentence is *not valid* in the description logic of Brachman and Levesque Chapter 9:

$$[AND \ [ALL \ r_1 \ [AND \ d_1 \ d_2 \ d_3]] \ [FILLS \ r_1 \ c_1] \ [FILLS \ r_1 \ c_2]]$$
$$\sqsubseteq [AND \ [ALL \ r_1 \ d_1] \ [EXISTS \ 2 \ r_1]]$$

For this question, your proofs should use the definition of validity and refer to interpretations. Use the notation in Brachman and Levesque Chapter 9.

3. [20 points] Use the tableau method for description logic $\mathcal{ALC}$ described in Baader and Sattler's paper to check whether the following concepts are satisfiable/consistent. Show the steps and rules that are used. If the concept is satisfiable give the model (satisfying interpretation) obtained by the method.

   a) $(\forall R.\forall R.\forall R.\exists R.\neg D) \sqcap (\forall R.\forall R.\exists R.C)$
   $\sqcap (\forall R.\exists R.B) \sqcap (\exists R.A) \sqcap \forall R.\forall R.\forall R.\forall R.D)$

   b) $(\exists R.A) \sqcap (\forall R.\exists R.(\exists R.A \sqcup \exists R.\neg B))$
   $\sqcap (\forall R.\forall R.\exists R.C) \sqcap (\forall R.\forall R.(\forall R.\neg A \sqcup \forall R.B \sqcup \forall R.\neg C))$

Bonus [5 points] This is a follow up to question 1. Use resolution with answer extraction to find the member of the Alpine club who is a mountain climber but not a skier. Show how the sentences obtained in question 1a are converted to clausal form and the resolution derivation. For each resolution step, state which two clauses are being resolved and the substitution used.