Computing for Math and Stats

Lecture 5

Matrix Operations

- Most regular arithmetic operations work for matrices and vectors
- You can add two matrices (or vectors)
  - As long as they can be added
- You can multiply them, too
  - Matrix and vector multiplication rules apply
- Division is a bit trickier

Division

- Our notation for real number division (fractions) rely on commutativity of multiplication
- In Matlab we need
    - Matrix inverse
    - Left division
    - Right division
- We have notation for all

Inverse

- We can invoke the inv() function
- We can raise a matrix to -1 (^-1)
- If the matrix is invertible it is inverted
- In an ideal world a matrix is invertible if its determinant is non-zero
- Back to reality, it is trickier
  - Computer accuracy is finite (due to round-off error)
  - The properties of matrices can play tricks (condition number)
- See invertibility.m

Left Division

- Solve AX = B
  - X and B are (column) vectors
  - A is square (and invertible) matrix
- Solution is X = A^{-1}B
- If we want to avoid the full inversion we write
  - X = A\B
- Matlab uses different numerical algorithms to calculate A\B

Right Division

- Solve XC = D
  - X and D are row vectors
  - C is a square (and invertible) matrix
- Solution is X = D C^{-1}
- In Matlab we write
  - X=D/C

Element-wise Operations

- These appear sometimes in linear algebra or statistics
- Appear often in image and audio processing
- Sometimes are useful shortcuts
- They have a dot before the regular operator
- We have .*, ./, .^

Built-in Functions

- Most functions that work on real numbers work on vectors/matrices as well
  - Sine, cosine, sqrt etc
- We also have
  - mean(A), median(A), sum(A), std(A)
  - sort(A)
  - m=max(A), [d,n]=max(A), min(A)
  - det(A), inv(A)
  - dot(v1,v2), cross(v1,v2)

Random Matrices

- Random vectors and matrices are extremely useful
  - To try our algorithms to see if they “always” work
  - Some algorithms require a random number generator (Monte Carlo methods)
- We have the
  - rand(m,n): uniformly distributed matrix. With one arg n produces n x n matrix and without it produces 1x1.
  - randi(imax, m, n): random integers 1...imax
  - randn(m,n): normally distributed