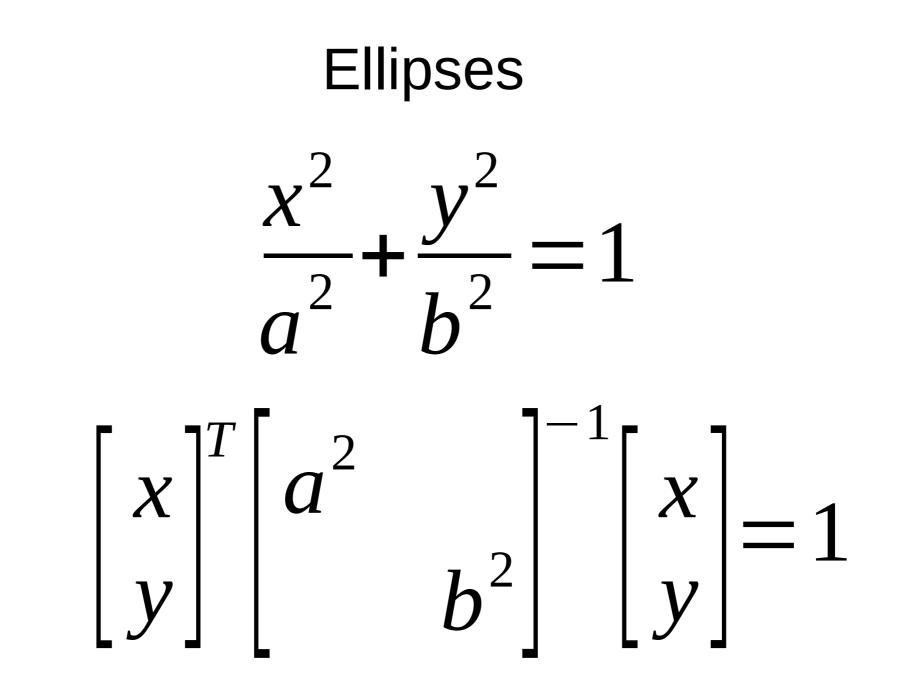
Computing for Math and Stats

Lecture 19



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^{T} M^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

$$\vec{x}^T M^{-1} \vec{x} = 1$$

$$(\vec{x} - \mu)^T M^{-1}(\vec{x} - \mu) = 1$$

- The equation of an ellipse can be written in a matrix form
- The matrix involved has to be symmetric
 If it is not we take the symmetric part of it.
- The matrix involved has to be positive definite
 If it is not it is a parabola or hyperbola
- Can be extended to higher dimensions
 - We can use the term ellipsoid for 3-D
 - We can stick the prefix hyper- for higher dimensions

- Drawing an ellipse
 - Create a set of points that satisfy the equation of an ellipse
- It is easy to take care of the center of the ellipse
 For now the center is at the origin
- We know how to draw a unit circle
 - We start from there

 $M = L^T L$

 $x_{c}^{T}x_{c}=1$

 $x = L^T x_c$

 $x^{T} M^{-1} x = (L^{T} x_{c})^{T} L^{-1} L^{-T} L^{T} x_{c} = i$ $x_{c}^{T}LL^{-1}L^{-T}L^{T}x_{c}=x_{c}^{T}x_{c}=1$

- Here is how we draw the ellipse:
 - Create the points to draw a circle
 - Multiply these points by the transpose of matrix L
 - Which we get by decomposing the matrix M
 - The resulting points form an ellipse
- The same exact procedure can be used for 3-D ellipsoids (or higher but then we cannot plot them)

- This procedure
 - Makes drawing easy
 - Given the matrix we can draw the ellipse
- But
 - Cannot draw hyperbolas/parabolas
 - The Cholesky decomposition does not work for matrices representing hyperbolas