Computing for Math and Stats

Lecture 15.

Multiplying Polynomials

$$\sum_{k=0}^{M+N} c_k x^k = \left(\sum_{i=0}^{N} a_i x^i \right) \left(\sum_{j=0}^{M} b_j x^j \right)$$

$$c_k = \sum_{i=max(0,k-M)}^{min(k,N)} a_i b_{k-i}$$

Multiplying Polynomials

- We now have to translate this to Matlab
- Vectors in Matlab start at 1
- Polynomial indices start at 0
- There is one more little problem
  - Matlab coefficients are in the opposite order
  - We ignore it and pretend that we know nothing
- The function that does this is conv
- See myconv.m

Add Polynomials

- We can add polynomials simply by adding the coefficients of same degree
- In Matlab the first element is the highest degree coefficient
- We need to align them by zero-padding
- See polyadd.m

Evaluating Polynomials

$$p(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$p(x) = ((a_3 x + a_2) x + a_1) x + a_0$$

Fitting Polynomials

- We are given a bunch of points (x, y) and we are asked to find a polynomial that goes through them
- We “evaluate” the polynomial at each x and equate the value to the y.
- From every point we get one equation
- We need N+1 points to fit an N degree polynomial

Fitting Polynomials

$$(x_0, y_0)$$

$$(x_1, y_1)$$

$$(x_2, y_2)$$

$$\vdots$$

$$(x_N, y_N)$$

Fitting Polynomials

$$a_0 x_1^0 + a_1 x_1^1 + a_2 x_1^2 + \cdots + a_N x_1^N = y_1$$

$$a_0 x_2^0 + a_1 x_2^1 + a_2 x_2^2 + \cdots + a_N x_2^N = y_2$$

$$a_0 x_3^0 + a_1 x_3^1 + a_2 x_3^2 + \cdots + a_N x_3^N = y_3$$

$$a_0 x_4^0 + a_1 x_4^1 + a_2 x_4^2 + \cdots + a_N x_4^N = y_4$$

$$\vdots$$

$$a_0 x_{N+1}^0 + a_1 x_{N+1}^1 + a_2 x_{N+1}^2 + \cdots + a_N x_{N+1}^N = y_{N+1}$$

Fitting Polynomials

$$\begin{bmatrix} x_1^0 & x_1^1 & x_1^2 & & x_1^N \\ x_2^0 & x_2^1 & x_2^2 & & x_2^N \\ x_3^0 & x_3^1 & x_3^2 & \cdots & x_3^N \\ \vdots & \vdots & \vdots & & \vdots \\ x_{N+1}^0 & x_{N+1}^1 & x_{N+1}^2 & & x_{N+1}^N \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{N+1} \end{bmatrix}$$

Polynomial Fitting

- What happens if we have more points
- This is very common in science and engineering
- The method we follow is called *Least Squares*
- One of the most important methods ever conceived by humans
  - Second only to the pizza recipe

Things to play with at home

- Modify myconv so that the indexing is proper. That is, there is no -1 in lines 6 and 7, indexing of the for loop starts at 1, etc
- Write a function named matlabpoly that accepts a vector p as argument (the coefficients of a polynomial) and prints a matlab expression that represents the polynomial.
- Write a function that accepts as arguments a vector p (the coefficients of a polynomial) and two numbers xmin and xmax and polts the polynomial p from xmin to xmax.