Computing for Math and Stats

Lecture 14.

Polynomials

- Polynomials are used extensively in science and engineering
  - Graphics
  - Signal Processing
  - Computer Vision and Image Processing
  - Cryptography
  - Statistics

Polynomials

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

Polynomials

- The degree of this polynomial is n
- The independent variable is x
- The a's are the coefficients
- The roots are the solutions to the equation $f(x)=0$

Representing Polynomials

- In Matlab polynomials are usually represented by the vector of the coefficients.

- The coefficient of the highest degree monomial is the first element of the vector

- The order of the coefficients is (obviously) important

- Most operations use this representation

Polynomials

- A polynomial can be represented by its coefficients
- Alternatively it can be represented by its roots and the first coefficient (the coefficient of the highest degree monomial)
  - See rootsdemo.m, rootplot.m
- Alternatively-alternatively can be represented by the values it has at a certain set of points
- For a polynomial of degree n we need n+1 numbers to represent it
- We can go from one representation to another
  - It is not always easy

Evaluating polynomials

- Matlab has the built-in function polyval
- Accepts as argument a polynomial and a value
- If the value is a vector it produces a vector of the corresponding results
- For large degree polynomials it can be tricky
    - If $x=9.5$ and the degree is 15, then we add together small numbers and huge numbers. This can create round-off errors.

Plotting Polynomials

- We can plot a polynomial using vectors

- We can also plot them using fplot
    - fplot(@(x)polyval(pp,x),[-3,3])

Roots of Polynomials

- Matlab has the function roots
- Accepts a polynomial as argument
- Gives back the roots of the polynomial
- It has to solve a very non-linear equation
- Function poly goes the other way:
  - Given the vector of roots it can compute the coefficients
  - This is much easier