## Computer Science and Engineering 4422.03/5323.03

**Test** Oct 26 2011

## Answer all questions in the space provided

Make sure that you have 9 pages

Student Last Name: \_\_\_\_\_

Student Given Name: \_\_\_\_\_

Student Id. No: \_\_\_\_\_

Question	Value	Score
1	60	
2	20	
3	55/65	

**Question 1.** [60 points]

1. [5 points] What is the main difference between distortion and aberration?

2. [5 points] What are the two scientific reasons that astronomers prefer large diameter telescopes? (consider the telescope as an expensive camera that observes far away objects)

3. [5 points] What are the two fundamental properties of the rotation matrix?

4. [5 points] How many parameters are needed to fully specify rigid motion?

5. [5 points] What is the most popular method to solve over-constrained linear systems with noisy parameters?

6. [5 points] What is the most popular function in computer vision? (at least for the stuff we have done so far)

7. [5 points] Name two robust methods that we covered in class.

8. [5 points] What is the name of the photosensitive surface of the human eye.

9. [5 points] What is the dimensionality of the Hough space for line grouping.

10. [5 points] What vector x maximizes the following Rayleigh quotient

$$\frac{x^T A x}{x^T x}$$

assume A is a symmetric matrix.

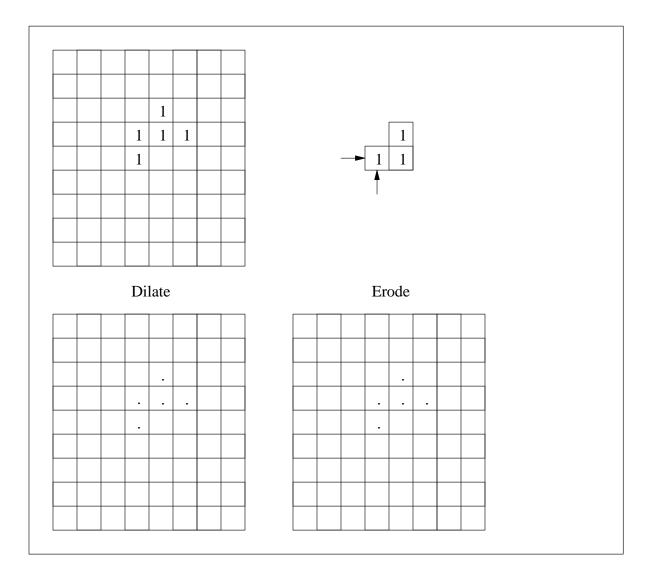
11. [5 points] What is the gradient of an image?

12. [5 points] A 3-D point has coordinates *X*, *Y* and *Z*. What is the homogeneous coordinates vector for this point?

## Question 2.

[20 points]

1. [10 points] Erode and dilate the following image with the template below and show the result on the empty "image". Pixels not marked otherwise are zero. The origin of the template is indicated by the arrows.



2. [10 points] A camera has resolution  $100 \times 200$  and CCD chip size  $10mm \times 20mm$ . The focal length of the camera is 20mm and the lens is directly in front of the CCD chip.

A. What is the calibration matrix that converts image coordinates to camera coordinates.

B. What is the inverse of that?

## **Question 3.**

[55 points]

1. [15 points]. You are given a set of N lines  $(\alpha_i, \beta_i, \gamma_i)$  for i = 1..N such that a point (x, y) belongs to line *i* if it satisfies

$$a_i x + \beta_i y + \gamma_i = 0$$

Devise a simple least squares algorithm, similar to the one one we did for ellipses, that finds the point closest to all of the lines. In particular

- (1) What are the knowns and unknowns and what is the minimum number of lines we need to solve the problem in 2-D
- (2) What is the expression we need to minimize?

(3) How do we minimize it and why we do it this way?

2. [20 points] We have to design a system that detects circles in an image. A typical image taken in the environment we work has about 10 circles of similar size and about half the edgels we detect belong to our circles and the rest are other edges of no interest to us. We will apply RANSAC. Assume that there are "many" edgels in the image.

(1) What is the probability that the first random point we select belongs to a circle. What is the probability that the second random point belongs to the same circle. What is the probability that the third random point belongs to the same circle. What is the probability that all three belong to the same circle?

(2) If  $w_3$  is the probability that all three belong to the same circle, what is the probability that we fail to find a triplet that belongs to the same circle after K trials.

3. [20/30 points] Assume an 1-D image *I* and that we want to fit a line to the intensity at points  $I[k - J] \cdots I[k + J]$ . The line will be of the form

$$\alpha j + \beta$$

and we would like to approximate the intensity as

$$I[k-j] \approx \alpha j + \beta$$

where  $j = -J \dots J$ .

(1) What is the mathematical expression we have to minimize

(2) Solve for  $\alpha$  and  $\beta$  and write the expressions as convolutions.

(3) GRADS: It is not surprising that the coefficients of a line fitted to the image is a convolution. Show that the coefficients of a quadratic fitted to the image are also convolutions.