

Homework Assignment #9
Due: Wednesday, December 9, 2020 at 5:00 p.m.

1. In class we defined (or will define) ZPP as follows. A language L is in ZPP if and only if there is a polynomial p and a probabilistic TM M such that for all strings x ,

- the expected running time of M on input x is at most $p(|x|)$,
- if $x \in L$ then all terminating executions of M on input x output “yes”, and
- if $x \notin L$ then all terminating executions of M on input x output “no”.

We can define another class ZPP' as follows. A language L is in ZPP' if and only if there is a polynomial p and a probabilistic TM M such that, for all strings x ,

- every execution of M on input x halts within $p(|x|)$ steps,
- if $x \in L$ then each execution of M on input x outputs either “yes” or “I don't know”,
- if $x \notin L$ then each execution of M on input x outputs either “no” or “I don't know”, and
- the probability that M on input x outputs “I don't know” is at most $\frac{1}{2}$.

Show that $\text{ZPP} = \text{ZPP}'$.

2. **Question 2 is for EECS5115 students only.** Define ZPP'' as follows. A language L is in ZPP'' if and only if there is a polynomial p and a probabilistic TM M such that, for all strings x ,

- the expected running time of M on input x is at most $p(|x|)$,
- if $x \in L$ then each execution of M on input x outputs “yes”, and
- if $x \notin L$ then each execution of M on input x outputs “no”.

Show that $\text{ZPP} = \text{ZPP}''$.

3. There is nothing to hand in now for this question. Read Section 2 of the following paper.

Russell Impagliazzo. A personal view of average-case complexity. In *Proc. of 10th IEEE Conference on Structure in Complexity Theory*, 1995. Available online via the York library at

https://ocul-yor.primo.exlibrisgroup.com/permalink/010CUL_YOR/sqt9v/cdi_ieee_primary_514853

You are not expected to understand all of this paper. However, you will be asked the following question on the final exam.

Which of Impagliazzo's five worlds would you prefer to live in? Write a paragraph to explain why. For full points, what you say about the conditions that exist in the world you choose should be true, and you should say at least one new thing about that world that is not mentioned in Impagliazzo's paper. (The new thing can be a small thing, but it should show that you have thought about the consequences of the assumptions made for the world you chose.)