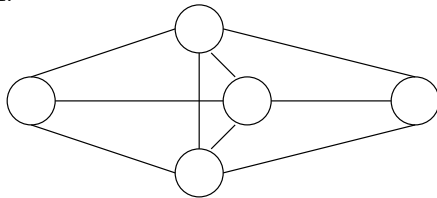


Homework Assignment #5

Due: Friday, November 6, 2020 at 5:00 p.m.

1. The chromatic number of a graph G , denoted $\chi(G)$, is the *minimum* number of colours needed to colour the nodes of G such that no two nodes connected by an edge get the same colour. For example, the following graph can be coloured with four colours, but cannot be coloured with three colours, so its chromatic number is 4.



In this assignment, we shall consider the CHROMATIC problem:

Input: an undirected graph $G = (V, E)$ and a positive integer k

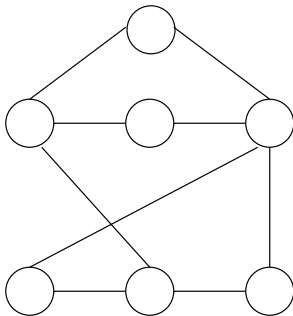
Question: is $\chi(G) = k$?

Recall the related 3-COLOURABILITY problem, which can be formulated as follows:

Input: an undirected graph $G = (V, E)$

Question: is $\chi(G) \leq 3$?

- (a) What is the chromatic number of the following graph? Explain why your answer is correct.



- (b) Show that CHROMATIC is NP-hard.
- (c) Show that if CHROMATIC is in NP, then 3-COLOURABILITY is in coNP.
Hint: One way to do this is to use the assumption to describe how to verify a certificate that a graph *cannot* be coloured using 3 colours.
- (d) Show that if $\text{NP} \neq \text{coNP}$, then CHROMATIC is not in NP.