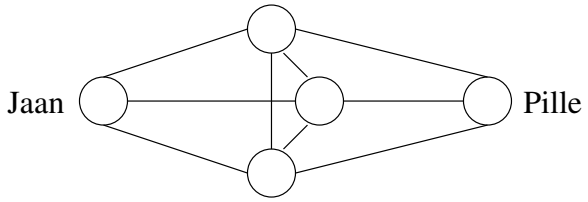
Homework Assignment #4
Due: Friday, October 30, 2020 at 5:00 p.m.

1. Suppose there are n children who must be divided into two teams, team 0 and team 1. Some pairs of children know each other. If any *three* children who all know each other get placed on the same team, they will waste too much time chatting. Is it possible to split the children into teams so that no team has three mutual friends?

More formally, the 3-CHATTY-CHILDREN (3CC) problem can be formalized as follows.
Input: An undirected graph $G = (V, E)$.
Question: Is there a subset $S \subseteq V$ such that S contains no 3-clique and $V - S$ contains no 3-clique?

(a) Show that 3CC is in NP.

(b) Show that in the following friendship graph G_1 (called a triangular bipyramid) it is possible to split the children into teams, but Jaan and Pille must be placed on the same team.



(c) Design a small friendship graph G_2, such that it is possible to split the children into teams, but two particular children, Tiiu and Kaja, must be placed on opposite teams. Explain why your answer is correct.

(d) Read about the ≠SAT problem, which is defined and shown to be NP-complete in Exercise 7.26 of the textbook.

Prove that ≠SAT $\leq_P$ 3CC.

(e) Consider the 2-CHATTY-CHILDREN (2CC) problem.
Input: An undirected graph $G = (V, E)$.
Question: Is there a subset $S \subseteq V$ such that S contains no 2-clique and $V - S$ contains no 2-clique?
Explain why 2CC is in P.